

# UNIVERSITY OF IOWA

# Studies in Psychology

EDITED BY  
GEORGE T. W. PATRICK  
PROFESSOR OF PHILOSOPHY

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## VOLUME III

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PUBLISHED BY THE UNIVERSITY  
IOWA CITY, IOWA  
1902

PRICE, ONE DOLLAR

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# UNIVERSITY OF IOWA STUDIES IN PSYCHOLOGY

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*Volume I. Issued in 1897. Price Fifty Cents*

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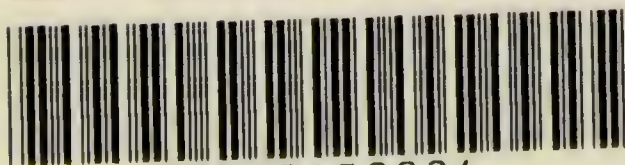
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*Volume II. Issued in 1899. Price One Dollar*

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The UNIVERSITY OF IOWA STUDIES IN PSYCHOLOGY is a bulletin devoted to the publication of researches carried on in the psychological laboratory. For the past two years the work of research in the laboratory has centered about the study of illusions in geometrical forms. Some of the results of these researches are reported in the present volume (Vol. III) in the article by Miss Williams entitled Normal Illusions in Representative Geometrical Forms, the basis of a work to be submitted by her as a thesis for the doctor's degree. Further experiments upon the same subject are reported by Dr. Seashore and Miss Williams in an article entitled An Illusion of Length. This is the only article in the present volume that has been previously printed.

The planning and building of a new laboratory, also described in this volume, has made it necessary to give much time and attention to the matter of equipment. This work has been under the immediate direction of Dr. C. E. Seashore, who in two articles describes some of the more important pieces of apparatus devised by him during this period.

*Iowa City, May, 1902*



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# A METHOD OF MEASURING MENTAL WORK: THE PSYCHERGOGRAPH<sup>1</sup>

BY  
C. E. SEASHORE

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In his presidential address at the meeting of the Psychological Association a year ago, Professor Jastrow outlined some problems in psychology that await solution at the present time. One of those problems is in part the subject of this paper, and I quote the following general statement of the needs, the possibilities, and the significance of this work:

“An adequate set of tests of normal functional efficiency, that shall receive a considerable authoritative sanction, is a great desideratum for the present-day needs, and an end by no means beyond the goal of properly directed endeavor. Its starting point is a correct analysis of the most distinctive modes of exercise of the several elementary components of our mental functions; the second step is the devising of tests that shall most simply, naturally, and definitely measure the functional efficiency of a selected factor or process; this accomplished, the way is prepared for the extensive utilization of such standards or norms of efficiency, (a) by their correlation with one another, (b) by a comparison with similar results obtained upon children at different stages of their development, thereby gaining an insight into the order and nature of their generic unfoldment, and (c) by a comparison with irregular, undeveloped, defective and decadent forms of such processes as they occur in connection with individual variation, with the

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<sup>1</sup> Read before the American Psychological Association Jan. 1, 1902.

consequence of mental stimulation, or in disease. This program, which could readily be expanded, is even in outline a most extensive one—rich in detail, fertile in mutual suggestiveness of its parts, possibly momentous in its practical consequences. The conclusion is obvious that for a host of comparative purposes the determination of norms or standards of functional mental efficiency is indispensable. That such determination involves conventions and artificialities is true and proper and inevitable. But neither is a foot, nor a meter, nor a candle-power, nor a volt, nor an ohm a natural and predestined *ding-an-sich*. Yet the arbitrary and conventional character of these units does not interfere with their utility. I am not advocating a ready-made mental yard-stick which shall show in what measure all men are not equal, and how each may discover the thumb marks of his individual success or failure. All this has been attempted before, and with necessarily futile results. The problem is recognized to be one of a general statistical nature, freighted doubtless with practical consequences, but the application of which must always be uncertain and dependent for its success upon judgment and insight.”<sup>1</sup>

These functional tests may be divided into two classes. To the first class belong the tests on single acts, such as an act of discrimination, memory, or voluntary control. The commonest of these are tests on the senses. To the second class belong tests on the repetition of one or more such processes, with continuous effort and without interruption, for seconds, minutes, or hours. Such continuous exertion of effort is commonly called mental work, and the amount of work done is estimated in terms of the results accomplished. The tests which I shall describe belong to this second class.

What is mental work, and in what sense can we speak of

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<sup>1</sup> JASTROW, *Psychol. Rev.*, 1901, VIII, 12.



measuring it? These two questions are naturally suggested by the title. Work is usually defined in terms of physics. By analogy we naturally think of mental work in terms of mental energy but an attempt to define this would lead us into endless difficulties. I therefore simply beg leave to use the expression mental work in the popular acceptance of the term, without involving any metaphysical implication concerning the expenditure of energy or any explanation of the causal relation between mental and physical processes. In the same manner, I must use the term measure in the freest sense. The idea of quantity is involved when we say a man solved ten problems, learned thirty verses of poetry, or settled five cases of dispute in an hour. But we should hardly speak of measuring these results, because the common units are too indefinite. The problems were not equally difficult and the man profited by practice; the verses differed and were not equally well learned; and the cases involved many variables. The determining of the amount of such work has ordinarily no scientific value because we cannot define the processes or control the conditions. We may speak of measuring the results of an act in proportion to the extent to which it is well defined and constant, and the conditions are known and controlled. The following description will show in what sense this can be accomplished. In professing to measure mental work, I simply attempt to answer with tolerable accuracy four questions in regard to the mental working capacity under given conditions: What? How much? How well? With what variations?

In dealing with this problem we are yet on the stage at which we must labor to perfect methods and means of measurement. The real problem will lie over, and many new ones will arise, while the experimenter devises tests and engages in a critique of method. Good progress has been made in the development of the first class of functional tests, but I am not aware of any generally accepted

test for the second class. Nor dare I hope that the principles herein set forth will prove a panacea. During the last three years I have devoted some time to the study of this principle of measuring mental work. One of the results is the elaboration of a principle of measurement and the devising and testing of apparatus for the purpose. The apparatus has received the descriptive name *psychergograph*, because it is a means of measuring mental work in the sense indicated above. In order to make the description clear and concise it will be necessary for me to make direct and unqualified statements and beg in return of the psychologist to supply self-evident qualifications.

In the designing of the psychergograph, I set myself the following aim: To devise means by which it shall be possible (1) to call forth a relatively simple and definite complex of mental activity, (2) to repeat the same for any desired length of time without interruption, and (3) to measure (a) the amount of work done, (b) the time taken, (c) the quality of the work, and (d) fluctuations in speed and quality.

A simple case of discriminative action seems best to serve as object of measurement. As a typical setting, I adopted the following: *Given one of four known signals, to recognize it and make the corresponding one of four simple responses.*

The apparatus consists of two distinct parts, the stimulator and the recorder. The stimulator makes a series of quick exposures of different signals. The observer is required to respond to each signal by a selective reaction. Every reaction brings out a new signal. The recorder makes a permanent graphic record. The two parts of the apparatus are seen in Fig. 1, the stimulator to the left and the recorder to the right.



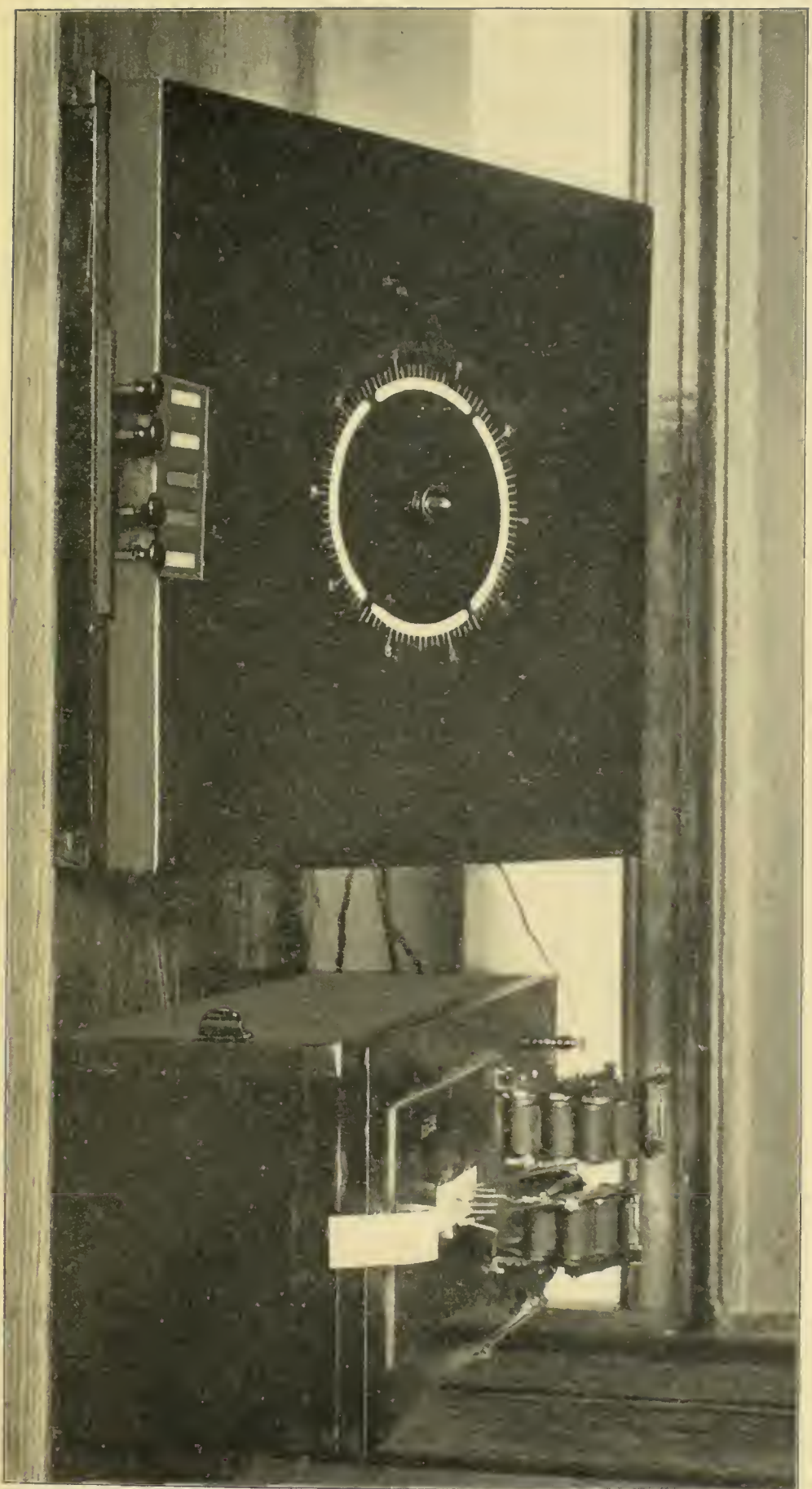


FIGURE 1. THE PSYCHERGOGGRAPH





The stimulator is seen as a plain case, 40 cm. square, with a slanting cover. Near the front edge of the cover is a signal window, 8 mm. wide by 20 mm. long, through which the signals are seen. One hundred signals are pasted or printed on a paper disk, 38 cm. in diameter, so that when the disk revolves they are seen singly in succession right back of the signal window. The paper disk is clamped on a metal wheel which has fifty teeth on its edge. This wheel is energized by a strong clock spring which revolves it and the disk one hundredth of a revolution, thus exposing a new signal, every time the detent which holds it is released. This detent is in the form of a lever escapement and is operated by electro-magnets. The manner in which it operates will be understood after the reaction mechanism has been traced. Four reaction keys are seen back of the signal window. There is a signal index on the surface in front of each of these. When a new signal appears, the observer responds by pressing the key indicated by the signal, and the slightest movement of the key instantly causes the next signal to appear. The connection between the key and the signal disk is electrical. There is one pair of electro-magnets on each side of the escapement detent. These are connected, through a battery, one with each side of a small rotating commutator. The commutator is in turn placed in circuit with a battery and the series of keys. Whenever a key is touched, this circuit is broken and the commutator turns one notch, thereby switching the current from one side of the detent to the other. This alternating of the current causes the detent to oscillate and allow the signal disk to move forward by one signal space for every movement of a key.

A circle of the revolving disk is seen through the cover. On this there is a cross line (at 43 in Fig. 1), which passes before the circular scale of a hundred units and indicates to the experimenter which signal is in view. This indicator serves at once as a counter of the number of acts

performed and as a guide for the beginning and the ending of a series. The order of the signals is determined in the making of the series, either by chance or by some suitable system. The experimenter, therefore, knows the actual sequence of the signals in every series, but the observer has no means of knowing at any time what signal will appear.

The spring is wound by a detachable key that fits on the projecting axis. All the electrical connections are made through binding posts and switches on the back of the stimulator.

The recorder furnishes a continuous tracing of the action of each of the keys in the stimulator, and parallel to these, a time-line. The essential parts of the recorder are electromagnetic markers, a record tape, a motor, and an interrupter.

The case of the recorder is 40 cm. long, 20 cm. wide, and 17 cm. high. Five markers are built up into a system on the surface, connected with batteries, and furnished with lead tracing points which mark five parallel lines upon the tape. An interrupter, to give a time-line, is placed in the circuit of one of these markers. The form and frequency of the interrupter depends upon how fine detail is wanted in the record. For seconds or half-seconds, some form of pendulum such as metronome or clock contact may be used; for tenths of a second the contact breaker made by the Cambridge Instrument Company may serve. Each of the other markers is connected electrically with a key in the stimulator so that whenever a key is pressed the circuit through that key is completed and the connected marker makes a jag in its line, thereby showing which key was pressed and the time relations of this act. Each marker is capable of three adjustments by means of thumb screws. The connections between the keys and their respective markers, and between the interrupter and its marker are made through the binding posts

on the top of the recorder and the back of the stimulator. The four markers have a common terminal and a single battery. There is a place for dry-cells inside of the case and either these or outside batteries may be used for the two circuits.

The record is made on common paper telegraph tape an inch wide. From a suspended spool in the case, it comes to the surface through a slit and is drawn under the writing points at a constant rate by a strong clock-work motor. The motor draws the tape by means of friction rollers and feeds it out free and in sight as it is shown in the Figure. In the present model the speed of the motor cannot be varied through a sufficiently large range. There should be three possible speeds, adapted respectively to the second, the tenth of a second, and the hundredth of a second reading. The motor is started and stopped by means of a button on the side of the recorder not seen in the Figure.

The apparatus is built in two pieces, in the first place, because it may be desirable at times to have them in separate rooms in order to reduce the disturbance. In the second place, this recorder—a multiple recorder—is useful for many other purposes in the laboratory. It is much more convenient than the kymograph in most cases of chronographic records in which the amplitude of the vibration is not an element in the measurement. In this respect it presents many points of advantage over the ordinary chronographs. Then, again, the stimulator may be used to advantage without the recorder in certain experiments. The total time of a series of acts may then be taken with a stop-watch and the trained experimenter can see directly whether any mistakes are made and what they are. The stimulator is then properly called a psychergometer or psychergoscope, as the amount of work done is read off directly by sight. This method can be employed only in certain crude forms of experiments.

Only a general scheme of the mechanism and the action



of the apparatus has been given. Its operation is simple. When a key is touched, a signal appears; the observer touches the corresponding key, and the marker connected with this key makes a jag in its tracing; the pressing of this key calls forth the next signal, and this process may be continued as long as one may desire. If the wrong key is pressed, the jag will occur in the wrong line. If there is any delay or interruption, this is shown by reference to the time-line. If there is any fumbling that will be shown by the duplication of jags.

From the experimenter's point of view, the operation of the apparatus is completely automatic. He has only to start the recorder by pressing a button and give the signal to begin. The personal equation of the experimenter is therefore completely eliminated. The records are permanent and may be read at leisure.

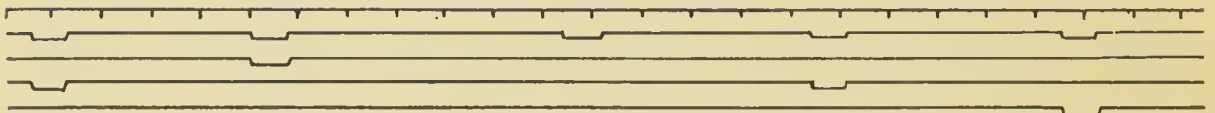


FIGURE 2. SECTION OF A RECORD

A section of a record is exhibited in Fig. 2. The upper line is the time-line divided into tenths of a second. The remaining four tracings are made with the markers so connected that whenever a key is pressed both the marker connected with that key and the marker next to the time-line respond. This double recording lends clearness and accuracy to the record. The time relations are all recorded on a single line, the one nearest to the time-line. The jags in the other lines simply show which key was pressed. The record represented in Fig. 2 shows that the keys were pressed in the following order: 3, 2, 1, 3, 4. Of course, when the first key is pressed there will be only one mark, as for the third act in the Figure.



A record of this kind is easily read. Take a long tape containing five hundred acts. It is first necessary to compare it with the standard series in order to trace the number and the nature of the errors. A list of the order of the signals is arranged as in the accompanying table, showing a standard record of a series of signals—or its equivalent, the required order of the responses. Here one hundred

## A STANDARD RECORD.

1	2	1	3	4	1	4	3	1	2
1	3	2	1	3	4	2	4	1	2
1	4	2	4	2	3	1	2	4	3
4	3	2	1	2	1	2	1	3	4
3	4	1	3	4	1	4	3	2	3
4	2	3	4	2	1	2	4	1	4
2	3	4	1	4	1	2	4	1	3
2	1	2	1	3	2	4	3	4	3
2	1	4	3	4	3	1	2	3	1
4	3	2	3	4	1	2	3	2	3

acts are grouped by fives and by tens. The grouping into fives is made in order to facilitate the comparison. In the table one may see the order of five acts at a single glance and then, by a glance at the record, determine the errors. Thus, the fifth line of the first section of the standard table reads, 3, 4, 1, 3, 4; but the corresponding section of the record, shown in Fig. 2, above, reads, 3, 2, 1, 3, 4. This shows that an error was made in the second act; the response was made to the second signal instead of to the fourth. Should this error occur with exceptional frequency, the cause of it may be readily traced.

In order to determine the fluctuation of working power in a long series of acts, I have found it convenient to divide the acts into groups of ten by a check mark on the time-line after each group in the first reading. This is facilitated by having the table in the present form.

Three kinds of time measurements may be obtained by examination of the tracing of the first key, which lies adjacent to the time-line. First, the time of the entire series is found by counting the number of seconds from the beginning to the end. Second, it is economical, and important otherwise, to compare the time of regular groups of acts. A specimen table of successive groups of ten acts each is appended below. This is a very good record. The

TIME OF SUCCESSIVE GROUPS OF TEN ACTS EACH.

*Observer, F. B. 497 acts. Total time, 296 sec.*

6.0	6.0	6.5	6.0	6.5	6.0	6.0	6.0	6.5 $e$	6.0
6.0	6.0	6.0	6.0	6.0	6.0	5.5	6.0	6.0	6.5
6.5	6.0	6.0	6.0	5.5	6.5	6.0	6.0	6.0	5.5
6.0	6.0	6.0 $e$	6.0	5.0	6.0	6.0	6.0	6.0	5.5
6.0 $e$	6.0 $e$	6.0	5.5	5.5	6.0	6.5	6.0 $e$	5.5	

observer shows remarkable self-control and power of endurance in continuous mental effort. The experiment was preceded by only two minutes of practice. The total time is only a little shorter than the average, but the constancy throughout the series is the distinctive characteristic. Only four errors were made. These are indicated by the small letter  $e$  for the groups in which they occurred. Most observers exhibit rhythmic fluctuations in speed and accuracy and characteristic variations with the progressive fatigue. Some work themselves into a "heat" and do better and better until some form of collapse occurs; others show progressive "disintegration" in loss of speed and accuracy, or in both. Third, the time of each individual act is obtainable. This may be taken in tenths or hundredths of a second and serves as a basis for the study of the variations within the group. In the specimen record

given, (Fig. 2), for example, the error in the second act caused a lengthening in the time of the third act.

What has thus been briefly described is the machine record. But every psychologist knows that this must be supplemented if any exact significance shall be ascribed to it. First, record should be made of the mental and physical condition of the observer before the test. The details of topics for these preliminary notes would depend upon the particular purpose in hand. Apparently trivial notes upon the previous and the present condition of the observer, if judiciously taken, may become complete explanations in the interpretation of peculiarities in the records.

Second, after each trial the observer must give an introspective account, as full as possible. He should especially try to account for the errors and describe the nature and the apparent causes of confusion states, periodic relaxation, progressive exhaustion, the struggle to inhibit automatisms, the intrusion of foreign trains of thought, and associations favorable or unfavorable. Experiments may be instituted for the express purpose of comparing the introspective account with the objective record.

Third, the experimenter has nothing to do but to watch the observer. He should make an objective study of conditions and effects, and should take private notes on the attitude of the observer in regard to interest, effort, strain, expressive attitudes, etc. He can make pencil checks on the record at the time to indicate parts to which the notes refer.

To pile up graphic records without some such supplementary notes as have been indicated would be sheer waste. The psychergograph gives us an accurate, unbiased record, but the real significance of this record depends upon our ability to account for the conditions which are elements in the process measured.

We may now turn to a brief outline of the possible vari-



ations in the experiments. The apparatus admits the introduction of a variety of signals. What the signal shall be depends upon the purpose of the measurement. It is often desirable to make the case as simple as possible. Then we reduce the difficulty of the discrimination to a minimum by making the signals as different as possible. We may take a color, a letter, a circle, and a dot. Or, we may take four readily distinguishable letters, geometric figures, pictures, words, or colors. In the test described thus far as typical, four clearly distinguishable colors were used. The degree of difficulty in the discrimination is a controllable variable and may itself be the object of measurement. Various degrees of small differences in the shade of colors, confusing geometric figures, or any other objects in which the degree of resemblance can be varied and described may be employed. Then again the higher processes in the act may be varied and complicated in many ways by requiring certain restricted associations, memories, judgments, and decisions based upon the recognized signal. In fact, all the variations and complications of the usual visual reaction experiment, both simple and complex, may be introduced, and provision is made for the uninterrupted and long continued repetition of the selected act.

The number of kinds of signals is not fixed. The apparatus makes it possible to use a hundred different signals, if so many should be wanted, as might be the case for example in a classification test in which the observer would be required to divide the whole series of objects into a small number of classes on the basis of certain class characteristics. But for most purposes it is best to use four signals, have an equal number of each kind, and have them of equal discrimination value.

Experiment has demonstrated that there is great advantage in using just four keys. The first model of this apparatus was made with ten keys. Unless one wishes to make the crudest kind of experiment, it is necessary to commit



to memory the order of the signal indexes before beginning the experiment. Ten proved entirely too confusing, because there was so great difference in the time that it took to find different keys, even after they had been well learned. Experiments were then made with six keys but even this was too many. Finally the system of four was adopted. In a series of discriminative actions like this, it is really necessary to have as many as four different ways of reacting in order to prevent anticipation, but very little is gained for this purpose by having more than four.<sup>1</sup>

The length of the experiment may be varied. The observer must work with a maximum degree of effort and there is no rest and no relief, because the work is uniform and continuous, requiring uninterrupted effort from beginning to end. Under these conditions a hundred acts is a great task and for many purposes that length of a series is satisfactory. But no one will learn the order of a hundred indifferent signals of this kind the first time he goes over them, especially if his effort be concentrated upon speed and accuracy. Furthermore, the observer has no means of knowing in what part of the series he begins or how he is progressing, because a small screen is placed in front of the scale. Therefore he may continue and go over the same series several times without learning to foresee a single sequence. Much depends upon what plan is fol-

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<sup>1</sup> In the discussion of this paper before the American Psychological Association, Professor Jastrow stated that it was not necessary to construct a special psychergograph because a typewriter may be used, and he recommended the Oliver machine on the ground that it prints the letters in plain view. So far as I am able to see, there are close restrictions to such use. Suppose, as was suggested, that the labels on the keys are interchanged by pushing lettered paper caps over the keys. If only a small number of keys should be used, the observer would soon learn this little series within which no variation obtains. If, on the other hand, a large number of keys should be used, it would take longer to learn the series but gross confusion would ensue in hunting for the keys. The difficulty is this: the typewriter key always prints the same letter throughout the series, whereas the psychergograph key may call forth any one of the total number of signals. This is an essential and radical difference. There are, however, many tests on routine processes that may be made with the typewriting machine.

lowed in the arranging of the series. If the order is determined by chance, two or more signals of the same kind might come in succession and these might serve as a clew to memory. Some order like that shown in the specimen series given on page 10 is better. If several tests are to be made in each sitting and to be repeated on several days, a different series of the same kind may be used for each test. But very many repetitions of the same series may be made without any disturbance from memory, if proper precautions are taken. In fact for some purposes, it is necessary to use the same series in a large number of successive trials.

The index signals are placed at the same level as the signal window and the keys, and they are all so close together that it is not necessary to move the point of regard from the signal window in order to see the index and the keys. The position of the four indexes are, however, learned before beginning the test and, for long experiments with the same order of signal indexes, it is best to have them covered and trust entirely to memory.

One important means of varying the experiment, as in the study of habit, consists in changing the order of the index signals.<sup>1</sup>

The bodily movement in the process, the pressing of the keys, is reduced to a minimum and a constant.

This outline of the construction of the apparatus and the method of measuring mental work is reported apart from its connection with any specific problem of research partly because, like the chronoscope or the kymograph, the psychergograph is adapted to a variety of uses. One investigation now in progress with this apparatus and method is on the subject of fatigue.

There are certain objections to this apparatus; for instance, its action is not soundless, it requires batteries, and it is rather expensive. But if the principle of the experi-

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<sup>1</sup> Some of these variations are suggested by Jastrow in his description of a sorting apparatus, *Psychol. Rev.* 1898, V, 297.



ment is correct, these objections may be overcome in time. This is the third model. The three have been built upon entirely different mechanical principles. In the first model, a column of celluloid disks was fed up to a signal window by a spring and at each reaction an electric hammer knocked away one disk, thereby exposing another. The same kind of record was obtained as in the present model, but through a simple mechanical action. This first model is so simple that it can be built in two days, by one man. I have used it successfully in more than a hundred experiments. For the one experiment, with colors, it is satisfactory, but it is limited to that use. The second model was a belt machine, the signals being carried on a belt which moved before the signal window. This proved unsatisfactory, probably on account of the imperfect workmanship in it. Thus there are many ways in which the main end may be accomplished. Then again, it requires only a minor step to make adaptations for the use of other sense stimuli, such as auditory or pressure stimuli.

The record may be called a psychergogram. It is truly a measure of mental work. First, it gives the amount of a particular work done. Second, it gives the time of each act, the time of small groups of acts as by fives or tens, and the total time of the series of acts. Third, it gives a quantitative expression to the quality of the work in terms of the number of errors, the sequence of errors, and the classification of errors. These are the mechanical elements in the record on the tape. Every feature of the record is a fact and these facts may be reduced to statistics, provided the conditions are observed and taken into account in the interpretation. He who wants the facts must labor to observe the conditions. There is a class of would-be observers who, when they see the beautiful automatic operation of the psychergograph, rejoice that they have at last found an automatic accumulator of statistics on mental processes. The psychergograph is not dedicated to them.

# A VOICE TONOSCOPE <sup>1</sup>

BY

C. E. SEASHORE

---

The study of motor processes has remained comparatively neglected in experimental psychology. It is not that the motor processes, as compared with the sensory, are less significant, of a baser sort, so much simpler, or fewer in number. It is because the analysis of the one naturally precedes the analysis of the other. The study has been pursued in the natural order in this inceptive stage of the science, but interest now begins to center upon the study of the motor process, not only because the motor process is the outcome of, and the sequel to the sensory process, but also because it is the practical phase of life.

In the psychology of music, the time has now come to begin to turn from the study of the hearing of tones to the study of the singing of tones. But, as we have had need of delicate tone-giving instruments in producing and gauging stimuli for the sensory processes, we now need, in addition, the corresponding means of measuring the results of motor processes. The lack of such instruments is conspicuous. The unaided ear will not serve the purpose. The graphic and photographic apparatus in use are too cumbersome. It is difficult to obtain a simple and at the same time accurate and ready means of measuring the pitch of tones produced by the human voice.

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<sup>1</sup> Read before the Section of Anthropology of the American Association for Advancement of Science, Jan. 1, 1902. The name "tonometer" was then used instead of tonoscope, but it is apparent that the latter term is preferable because it distinguishes this instrument more completely from other tone-measuring instruments.



The apparatus that I am to describe is somewhat complicated in construction, but when once built it is easy of manipulation, can be used for rapid work, and can be made sufficiently accurate for all purposes of routine measurements for which it is adapted. It is intended to be used in measuring the pitch of the human voice in singing and speaking; and, since it causes the vibration-frequency, which denotes the pitch of the tone, to be *seen* directly when the tone is sung, this apparatus may pass by the descriptive name of voice tonoscope.

The voice tonoscope is constructed on the principle of the stroboscope; that is, the vibrations of the voice are made visible upon a moving surface by the action of intermittent light. I take pleasure in acknowledging my indebtedness to Professor Scripture, who first employed this method of measurement in an exercise on the reproduction of tones.<sup>1</sup> The apparatus is "built up"; the individual parts, such as the manometric capsule, the vacuum tube, the stroboscopic disk, and the double contact fork, are used in physics, but Scripture was the first to use them in this way for psychological purposes. The special feature of the present apparatus is the stroboscopic screen, shown in Fig. 3, but a brief description of the complete apparatus and of the methods of measuring by means of it will be given.

The following are the essential parts of the apparatus: (1) means of producing a standard tone, (2) a special moving stroboscopic screen, (3) means of projecting the vibrations of the standard tone upon the screen, and (4) means of projecting in a similar manner the vibrations of the tone sung. These parts will be described in the order in which they have been mentioned.

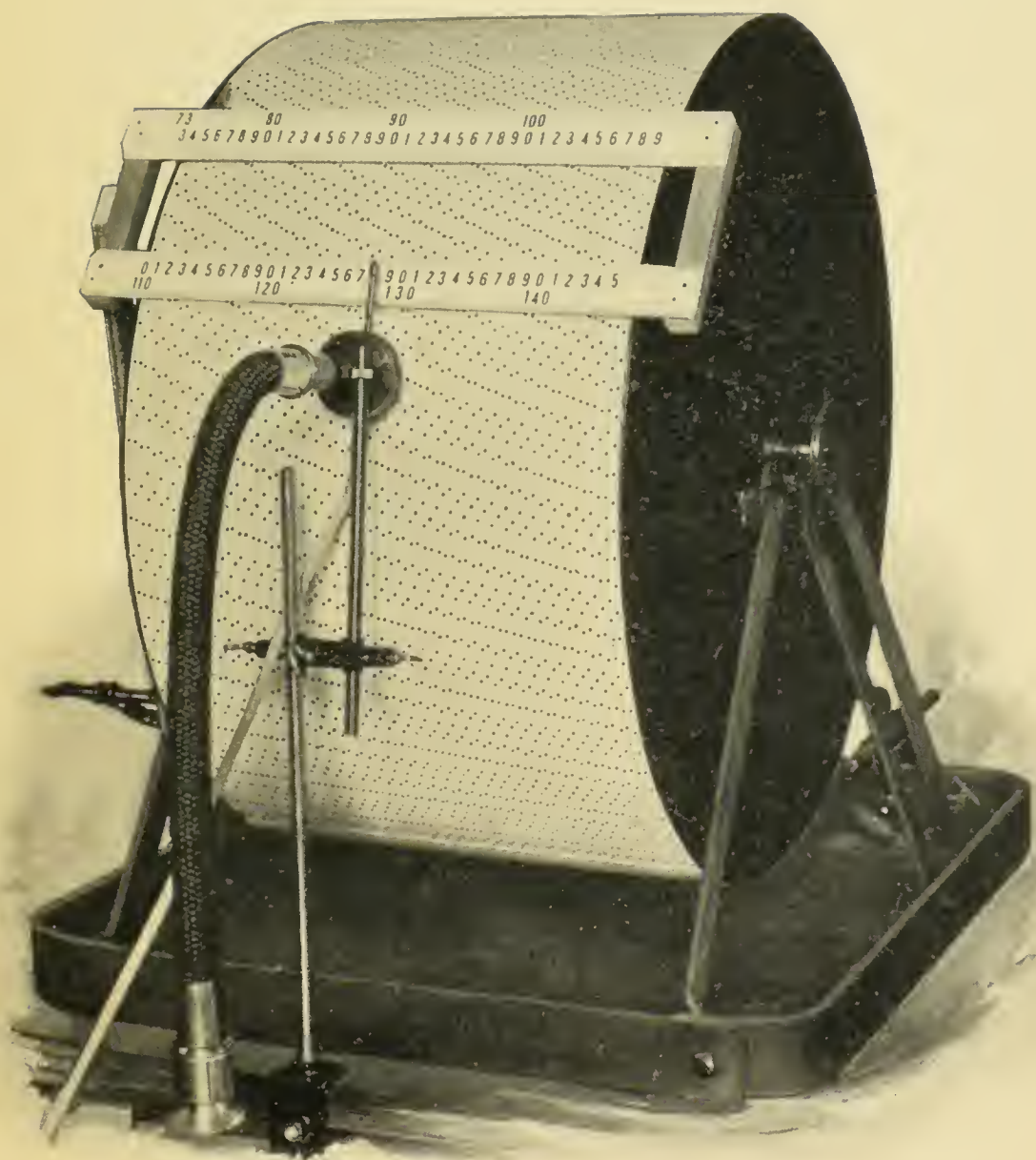
The standard tone, from which the singer takes his key

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<sup>1</sup> SCRIPTURE, *Elementary Course in Psychological Measurements*, Studies from the Yale Psychol. Lab., 1896, IV, 135.

and by which the standard of measurement is determined, is produced by an electric fork in a distant room and is heard through a telephone receiver which is in shunt circuit with the fork. A switch makes it possible for the experimenter to sound the standard tone at required intervals of time; or the observer may himself regulate this roughly by moving the receiver to and from the ear.

The stroboscopic screen is made in the following manner: A metal drum 50 cm. wide and 50 cm. in radius is mounted on ball bearings and revolved by an electric motor. A heavy white paper is stretched around the drum and on this paper seventy-one parallel lines of dots are drawn. The dots are 3 mm. in diameter and equidistant and the lines extend through the complete circumference of the drum. The lines of dots are divided into two series. The first series contains thirty-six lines drawn equidistant and so spaced as to cover the whole screen. Their arrangement and the number of dots in each is shown by the numbers in the upper section of the scale seen in the Figure. The first line has 73 dots and each succeeding line in the series has one more dot than the line immediately preceding. The second series has thirty-five lines and these alternate with those of the first series. This arrangement and the number of dots in each line may be seen by the numbers on the lower section of the scale. As regards the number of dots in each line, the second series forms a direct continuation of the first and the dot-frequencies of the two series together, therefore, extend from seventy-three to one hundred and forty-five by a uniform increment of one dot for each line. The aim in the arrangement of the dots was to secure dot-frequencies which should correspond to the vibration-frequencies of all tones within the range of the human voice. In musical terms, we have here what corresponds to the vibrations of all tones within the octave 73 v. d. to 146 v. d. The stroboscopic effect is such that the dot-frequency for one



THE TONOSCOPE





octave will serve quite as well for tones in higher or lower octaves, as will be explained later.

The standard tone is reproduced to vision upon the screen. The accessories for that purpose are not shown in the illustration. On the back side of the drum, there is a scale like the one seen on the front side and occupying a corresponding position. A vacuum tube, 30 cm. long, is fastened at the upper edge of the scale and supplied with a reflector which throws the light on the scale and prevents it from shining over the drum. The terminals of this tube are connected with the secondary terminals of an induction coil. The primary circuit of the coil is completed through a battery and the double contact fork that produces the standard tone. The room is darkened in order to secure the intermittent light effect. Each vibration of the fork breaks the circuit and the resulting spark produces a flash of light in the vacuum tube so that the screen is lighted up once for each vibration of the standard fork. Suppose that the fork is a 100 v. d. fork and the drum is revolving at the rate of one revolution per second. Then, when the dots on the revolving screen pass under the intermittent light, the line which has one hundred dots appears to stand still and is clear and well defined, because the frequency of the dots is the same as the frequency of the lights: every time an image of this line is cast upon the retina the dots appear in the same position. The dots in all other lines appear blurred and moving, assuming more or less the appearance of gray streaks. It is therefore easy to detect the line that is wanted. The purpose of this contrivance for projecting the standard tone is merely to secure the means of regulating the speed of the drum. Thus, in the present case, the speed of the drum is regulated so that the one hundred-dot line stands still continuously. This line then becomes the standard for the visual scale in terms of which the measurements are to be made. Any other tone projected on the same screen in a similar

way will make some other line stand still and the pitch of that tone will be indicated by the number of that line, or a multiple of it; e. g., if the second tone makes the 105-line stand still, it is a tone of 105 v. d., or some multiple of that number, as 210 v. d. or 420 v. d. Which octave it is in can readily be told by the practiced ear but it is also shown by the formation of the dots.

The tone that is sung is projected on the same screen, in a similar manner, although by different means. In this case the intermittence of the light is produced by means of a manometric flame on the side of the drum opposite the vacuum tube. The manometric capsule, with the connected gas tube and the speaking tube, is seen in the Figure. The observer holds the speaking tube before his mouth in the way such tubes are usually held, and sings the tone that is to be determined. The tonal vibrations cause sympathetic vibrations of the little gas flame in the capsule so that the flame rises and recedes once for each vibration of the vocal organs. This gives an intermittent light like the one produced in the vacuum tube on the other side of the drum. When the screen is moving at the standard speed, that line of dots will stand still which has dots to correspond to the frequency of vibration of the tone, and the pitch of the tone is indicated by the number of the line that stands still. The pitch may be seen as soon as it is heard because the visual and the tonal effects are synchronous.

This summary description of the essentials of the apparatus also indicates, in a general way, the mode of procedure in experimenting. It reduces itself to this: the singer takes the key tone from the receiver and sings a tone before the speaking tube; the vibrations are projected on the moving screen and the pitch of the tone is seen on the scale.

Although the apparatus consists of several parts, only the drum with its screen is made for this specific purpose.



The other parts are such as are found in any ordinary physical or psychological laboratory, and need not be described in detail, but some further particulars about the construction and use of the screen are necessary.

The arranging of the lines into two series is important. It is necessary to have all the lines within a certain compass because the necessarily small light will not serve for more than a limited space and the spreading of the record over a large surface would therefore interfere with quick reading. This drum was made as wide as is practicable under the named restrictions. An attempt was made to put the lines in a single series and it was found that then only about thirty lines could be put on the screen and be read satisfactorily. This number of lines would cover only a part of an octave within the desired range of tones. At least seventy lines are needed. By the double series plan it is possible to put seventy-one lines on the space occupied by thirty in the single series plan; and it is easier to read on this screen of seventy-one lines than it was on the original screen of thirty lines covering the same space. The reason for this lies in the fact that by the double series plan no two adjacent lines have nearly the same dot-frequency; consequently the line on each side of the one that stands still appears as a gray streak and forms a sort of frame for the line one is reading. This has two effects: in the first place, the two series may be placed on the same screen without causing any interference; in the second place, the one series of lines makes the other clearer, so that it is possible to put the lines closer together. Having separate scales for the two series is also conducive to clearness. The same device, therefore, makes it possible to save space and add clearness and thus to provide for the desired range of tones.

Let us suppose that the speed of the drum is regulated by the 100 v. d. fork to make one revolution per second. Then, if the tone that is sung lies within the range of the

octave from 36 to 72 v. d., the line that stands still has twice as many dots as there are vibrations in the tone and, in the reading, the number on the scale is simply divided by two; e. g., if the 75-line stands still the tone is  $37\frac{1}{2}$  v. d. The ear readily reveals whether or not the tone lies within the low octave and these low tones can be sung only by the male voice. For tones from 73 to 146 v. d., the reading is direct from the scale. This represents the middle range for the baritone. For tones within the octave from 146 to 292 v. d., the line that stands still appears to have twice as many dots as it actually has. That makes it easy to recognize this octave and, in the reading, the number on the scale is doubled: e. g., if the 75-line stands still, the tone is 150 v. d. This octave represents the upper range for the male voice and the middle and part of the lower range for the female voice. From 292 to 584 v. d., there will be four vibrations for each dot in the line that stands still and the numbers on the scale must be multiplied by four. These multiples may be placed in small figures on the scale. The same principle of reading may be extended to higher octaves in so far as it depends upon the screen, but there is a limit set by the degree of sensitiveness of the diaphragm in the manometric capsule.

If a high tone is studied, it is well to run the drum at the rate of two revolutions per second. This can of course be regulated by the 100 v. d. fork. Higher forks may also be used for the sounding of the key-note after they have been tuned with reference to the 100 v. d. fork. The tonoscope may be used in tuning them.

It is only relatively true that a line stands still; it stands still only when there is an absolute agreement between the vibration-frequency and the dot-frequency. If the vibration-frequency is a fraction greater, the dots move gradually downward, and if it is less, they move gradually upward—and the speed of the apparent motion is always in proportion to the difference. Thus, if the tone is 125.5 v. d., no



line stands still, for the 125-line moves downward at the rate of one space per second and the 126-line moves upward at the same rate; but, if the tone is 125.1 v. d. the 125-line moves downward at the rate of only one-tenth of a space per second and the 126-line moves upward at the rate of nine-tenths of a space per second. The practiced observer can therefore readily learn to read in tenths of the scale units.

The accuracy of the reading varies with the accuracy of the singing. The accuracy increases with the steadiness of the voice, and hence with the demand for accurate measurement. The more firmly the pitch is sustained, the easier is the reading. The ordinary singer wavers in the tone that he tries to sustain even for half a second and the experimenter must select the predominating pitch in it.

The drum is heavy and carefully mounted so that it runs at a fairly uniform speed, but if the speed should tend to vary, the experimenter records such variations and a corresponding correction in the readings is made from a table which is exact to one-tenth of a vibration.

The tonoscope is not constructed for any one particular psychological experiment. It is intended to be a general measuring instrument that may be employed in a number of ways. It is simply a quick and accurate means of determining the pitch of tones—not only of the human voice, but also of instruments from which the sound may be projected upon it. In coöperation with a student, Mr. Edward Bechly, I have begun a study of certain facts about the nature of the control of the voice in singing and speaking. The results are not ready for publication but some of the measurements that we have made may be mentioned in order to indicate some purposes for which the tonoscope has been used with excellent success.

1. Sounding a single tone. This measurement presents different aspects according as the tone is taken at random, is heard from a standard, or is heard from a standard and



hummed before being sung. The absolute pitch, the intensity, the mode of vocalization, and the time relations are factors that may be varied in order to determine their effect upon the pitch of the tone.

2. Sustaining a tone. "Sustained" tones vary in pitch according to a general law. The factors that influence this may be determined.

3. Singing tones at certain pitch intervals. The standard tone may be given or it may be taken at random by the voice.

4. Singing musical scales, natural and chromatic, with numerous variations in the conditions.

5. Singing a melody. There is a shocking surprise in store for every singer who tests his ability to sing even the simplest air in true pitch. He finds that his good ear has tolerated great license in the matter of fidelity to pitch. Two-part harmony, or unison singing, may also be measured by recording for one voice on each side of the drum, but this we have not tried.

6. Speaking in a uniform pitch (a pure fiction).

7. The least producible difference in pitch. This is the measurement that is of greatest value for psychology. The least producible difference is to the study of motor processes what the least perceptible difference is to study of sensory processes.

# AN ILLUSION OF LENGTH<sup>1</sup>

BY

C. E. SEASHORE AND MABEL CLARE WILLIAMS

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Two years ago, while studying the æsthetics of geometrical forms, we required the students to produce 'double squares' by direct eye estimation. The forms were produced by means of an adjustable frame and were to be made twice as long in the horizontal direction as in the vertical. Not only was the illusion of the vertical counteracted, but there appeared to be a tendency to make the horizontal distance too short.<sup>2</sup>

We repeated the same test with two hundred school-children,<sup>3</sup> ten boys and ten girls of each age from six to fifteen inclusive, and found that this illusion of length is much stronger for children than for adults, and that it decreases with the increasing age of the children. Thus, the children of six made the double square 28% too short, the 7's 24%, the 8's 25%, the 9's 14%, the 10's 10%, the 11's 11%, the 12's 11%, the 13's 10%, the 14's 8%, and the 15's 9% too short. These figures seem fabulous, especially when it is remembered that they represent the amount by which the well-known illusion of the vertical has been outweighed. The illusion of the vertical was measured in the same series and found not to vary noticeably with the age of the children.

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<sup>1</sup> Reprinted from THE PSYCHOLOGICAL REVIEW, VII., 592-599.

<sup>2</sup> "Forty-eight made the horizontal line too short by an average of 15 mm. (mean variation, 8 mm.), and fourteen made it too long by an average of 10 mm. (mean variation, 6 mm.). On the whole the horizontal distance was made 4½% too short." *Univ. of Iowa Stud. in Psychol.*, 1899, II., 18.

<sup>3</sup> *Op. cit.*, p. 33.












Continuing the study of this illusion in persons who knew little or nothing about the nature of the illusion, we made a series of measurements upon university students in introductory psychology, before the subject of illusions had been discussed in class. Twenty-nine men and thirty-four women volunteered to be tested. The so-called unconscious method was followed in so far as it was possible. Most of the observers, however, surmised that some illusion was involved in the test, and the conscious or sub-conscious reaction to such suspicion often entered into their judgments. The records of two men were discarded because these men asserted that they had attempted to eliminate the illusion of the vertical. The observers may evidently be divided into two classes, namely, those who introduce some correction for supposed illusions and those who do not. But as such division is relative and cannot be made satisfactorily either upon introspective testimony or upon inspection of the results, we group the records together and state some of the conclusions which may be drawn from the averages of the results.

The forms to be studied were cut out of cardboard and then placed on a neutral background upon a large drawing board, which was so placed that they were 50 cm. from the eyes and at right angles to the line of regard. Some of the forms were drawn on the background, as will be explained. Every effort was made to eliminate disturbing influences from color, brightness, limiting lines and spaces, movements in the adjustment of the cards and other known sources of error. The forms of figures may be grouped into five classes, as is shown in the accompanying table. They are: (A) parallelograms, (B) a vertical and a horizontal line, (C) and (E) two horizontal lines of different length, and (D) unequal horizontal distances limited by points.

In the A-series, rectangular forms were to be produced as follows: A1, a double square, double in the horizontal



direction; A2, a double square, double in the vertical direction; A3, a half square, the standard vertical; and A4, a square, the standard vertical. The standard for all was the side of a square, 114 mm. A card 114 mm. wide and about 275 mm. long was used, and the observer was required to produce the respective forms by placing a similar card over such portion of it that the remaining part would constitute the required form. The figures in the B-series are parallel to those of the A-series and consist of only two of the adjacent lines from each form of the parallelograms. They were to be produced as follows: B1, the horizontal line twice as long as the vertical; B2, the vertical line twice as long as the horizontal; B3, the horizontal line half as long as the vertical; and B4, the horizontal line equal to the vertical standard. The variable line was 275 mm. long, and the observer covered such portion of it with a card that the remaining part of it stood in

Form.											
Case St.	A1 228	A2 228	A3 57	A4 114	B1 228	B2 228	B3 57	B4 114	C 228	D 228	E 228
	X d	X d	X d	X d	X d	X d	X d	X d	X d	X d	X d
M	222 9	213 10	62 3	115 3	226 7	211 6	64 3	119 2	225 8	227 11	217 7
W	222 5	214 5	63 3	116 3	223 7	216 5	63 2	120 4	226 7	227 9	216 9
Ave.	222 7	214 7	63 3	115 3	222 7	214 5	63 2	120 3	226 7	227 10	217 8
% E.	-3 3	-6 3	+11 5	+1 3	-2 3	-6 3	+11 4	+5 3	-1 3	0 4	-5 4

*Form*, the general shape and the position of the forms.

*Case*, the designation of each case in this report.

*St.*, the required length of the measured line or distance. The unit of measurement is the millimeter.

*X*, the average of the results of the estimates: (M) by men, (W) by women, and (Ave.) by all together.

*d*, the averages of the mean variations.

*% E*, the differences between the estimate and the standard, stated in percentages of the required lengths. The mean variation is also reduced to percentages.

The number of trials is two for each person upon each point in Cases A, B and D, and ten in Cases C and E. The double fatigue order was observed.

the proper ratio to the standard line. In Case C the two lines lie in the same direction (horizontal) and the measurement was made as in the B-series. The variable line was placed 10 mm. lower than the standard in order to eliminate the motive to connect them. In Case D the standard distance (horizontal) was given by two points. The observer was required to place a third point so far to the right of these that the distance between the second and the third points appeared to be twice as great as the distance between the first and the second. In Case E, the figure is the same as in Case C, but the observer was required to select the limiting point before applying the limiting card.

The final averages of the results may be seen in the accompanying table. Case A1 represents the original form of the illusion. The double square is made 3% too short. This indicates that the illusion of length exceeds the illusion of the vertical by that amount, because the two illusions stand in direct opposition in this position of the figure. If only the illusion of the vertical had been present, the double square would have been made too long. Case A2 shows the effect of the coöperation of the two illusions. Here the double square is made 6% too short. We have found, in a large number of other experiments, that the two illusions in coöperation produce an illusion equal to the sum of the two illusions acting singly; still that is not always to be expected, because the effect of one illusion partly satisfies the motive of the other. Case A3 was introduced as a check upon the method of making the comparison and the measurement. In shape this figure is identical with A2, but it differs from it in size, and was produced by varying the shorter dimension. This half-square was made 11% too wide. In all our experiments it is found that when the vertical distance is varied there is a stronger tendency to correct the known illusion of the vertical than when the horizontal distance is varied. This is because more atten-



tion is called to the presence of that illusion by the former method. That law does not apply to the illusion of length, because the observers had no means of knowing that such an illusion existed. As will be shown later, the difference in size does not account for the difference in the illusion for the half-square and the double square. Case A4 contains only the illusion of the vertical. The horizontal distance is made 1% too long, but that is not a representative figure. Some four hundred measurements upon students gave an average of 5% for the illusion of the vertical. It is difficult to say how far this tendency to correct the illusion of the vertical enters into the estimation of the double square in the present experiments. It is probable that it increases the difference between the two illusions in A1, and that it decreases the resultant of the two in A2 and A3.

We conclude from the study of the A-series, then: (1) that the illusion of length obtains for both the horizontal and the vertical positions of the double square, (2) that it is stronger than the illusion of the vertical, and (3) that the tendency to make unconscious corrections for an illusion is greater when the image of the illusion is clearest in the mind.

The B-series was so arranged as to determine whether the illusion is a linear illusion or is peculiar to the comparison of the two dimensions of rectangular areas. The relations of the dimensions of a parallelogram may be estimated, (1) by comparing a vertical margin with a horizontal margin, (2) by comparing the length of imaginary lines that cross at the center of the figure, and (3) by judging by the total impression with reference to the areal content without distinctly selecting representative lines. The first method is the easiest and the most accurate, and was followed by nearly all our observers. When the second method is employed, the illusion is increased on account of the increased vagueness of the parts to be compared. The



third method is the most valuable and probably entails the greatest illusion. Children in their quick and spontaneous judgments follow this method, as a rule, and this accounts partly for the strength of the illusion with them. When the first method is employed we have essentially the same conditions of comparison in the A-series as in the B-series. The average of the results for B1, B2 and B3 are equal respectively to the results of A1, A2 and A3. B4 shows a normal illusion of the vertical.

From the comparison of the results of these two series of tests, we conclude that the described illusion of length in surfaces is essentially a linear illusion.

Is this linear illusion contingent upon the difference in the direction of the two lines compared? In Case-C both lines are placed in the same direction, and the illusion of the vertical is eliminated. All the results for lines lying in the same direction show that the illusion is present in such lines, but is very much diminished by the elimination of the difference in direction.

Case D was introduced in order to determine whether the illusion of distances in the same direction is contingent upon the presence of lines. The results are affirmative; there appears to be no illusion for the distance between the dots.

Case E was introduced in order to determine whether excessive eye movements constitute one of the motives for the illusion. In Cases A, B and C the comparison is naturally made by bisecting the long line visually, to determine whether one-half of the long line is equal to the length of the short line. When one-half of the line is regarded, the eye wanders beyond the middle point into the other half, on account of the guiding power of the line and the absence of a definite objective limit. This overrunning of the point of regard ought to produce an overestimation of the half that is regarded, and thus an overestimation of the whole line. Case E is like Case C in every respect except that there is

an additional motive for excessive eye movements in Case E; the eye is allowed to wander to the right beyond the limit to be selected. The results show that the introduction of this additional motive for excessive eye movements produces a marked increase in the illusion. Since this new eye-movement motive, which is present in the bisection of a line, is of the same nature as the additional one which increases the illusion in Case E, we conclude that it is one of the primary causes of the illusion of length. We may refer to this as the first motive.<sup>1</sup>

Contrast is a second primary motive which may be present and co-operate with the first motive. It is an application of the principle of relativity—a long line is compared with a short line, and, according to this principle, the long line should appear to be longer and the short line shorter than it really is. With young children this contrast is undoubtedly the strongest motive, because children have strong tendencies to overestimate differences. It has been demonstrated that illusions which have physiological causes, *e. g.*, the illusion of the vertical and the Müller-Lyer illusion, do not vary in a marked manner with mental development; but the illusions of judgment, *e. g.*, the illusions of contrast and illusions of time, vary very much with mental development.<sup>2</sup> Therefore it is probable that, if both the physiological and the psychical motives were present in those of the tests that were made upon children, the variation with age (mental development) is due chiefly to variation in the second motive, namely, contrast.

These two motives appear in the comparison of lines that

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<sup>1</sup> Incidentally we have here a crucial test of the eye-movement theory of the Müller-Lyer illusion. Cases C and E may be considered the simplest forms of the Müller-Lyer figure. All the motives demanded by current theories of this illusion, except one, are eliminated in these figures. The eye-movement motive is the only one present, and the illusion is present and varies with this motive.

<sup>2</sup> SEASHORE, *op. cit.*, pp. 33-35, 83, 84.



lie in the same plane and direction, but when the two lines lie in different directions or are parts of parallelograms, there appears in addition to these a third primary motive. It is a well-known fact that if a square and a double square appear side by side the latter will appear to be the narrower. This is not due to the contiguity of the two figures, for the dimensions of the double square do not appear to change when the square is removed.<sup>1</sup> Some would ascribe it to the principle of relativity, the two sides of the same figure being compared with each other. But it may be accounted for better by Wundt's theory of eye movements, *i. e.*, there is a stronger tendency to move the eyes in the direction of the longer lines than in the direction of the shorter.<sup>2</sup> This theory is in harmony with the most acceptable theory of the visual perception of space and is well supported by the fact that the total illusion of length is so much greater for parallelograms and lines at right angles to each other than for the lines that lie in the same plane and direction.<sup>3</sup> The results tend to show that this third primary motive, which has no connection with the first eye-movement motive, is, for adults, the strongest motive in the figures that contain it.

To determine the validity of the method employed, we made parallel measurements by the method of selection. The observers were allowed to select the required proportions from a series of cards. The results obtained by this

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<sup>1</sup> The theory that the apparent narrowing of the double square is due to an illusion of perspective is disproved by the fact that the figure does not appear to change when the square with which it has been compared is removed, and by the fact that the length of the figure is overestimated, instead of underestimated.

<sup>2</sup> Wundt, *Die geometrisch-optischen Täuschungen*, Abh. d. math.-phys. Cl. d. k. Sachs. Ges. d. Wiss., XXIV., 1899, 158.

<sup>3</sup> A part of this difference may be accounted for by the fact that the placing of the lines in different directions increases the difficulty of comparing and thus gives fuller sway to the motives for illusion that are present.



method support those obtained by the method of production as described above.

We also made a series of measurements by the method of selection to determine whether the ratio 2:1, employed above, is peculiarly favorable for the production of the illusion. The observers were required to estimate the length of cards in terms of their standard width. There appears to be a gradual increase in the illusion from its inception near the square up to the ratio  $2\frac{1}{2}:1$ , which was the largest ratio tried. The illusion is not relatively stronger for the ratio 2:1 than for the adjacent ratios.

Does this illusion vary systematically with the size of the figures? The averages of the results for four observers who made eighty trials each upon each of four sizes of cards of the A2 type above, by the method of production, are as follows: standard (length in the vertical direction) 76 mm., illusion 7%; standard 114 mm., illusion 8%; standard 228 mm., illusion 7%; and standard 456 mm., illusion 8%. The cards were in all cases 75 cm. from the eyes. There is therefore no constant tendency for the combined illusions to vary with the size of the cards. But it has been held that the illusion of the vertical alone increases with the size of the object. We therefore measured the illusion of the vertical for the corresponding sizes of squares. The same four observers made eighty trials each for each size of card, by the method of production, with the following average results: standard 57 mm., illusion 3%; standard 114 mm., illusion 2%; standard 228 mm., illusion 4%; and standard 456 mm., illusion 3%. Thus there is no constant tendency for the illusion of the vertical to vary with the size of the object, and therefore the same is proved for the illusion of length.

# NORMAL ILLUSIONS IN REPRESENTATIVE GEOMETRICAL FORMS

BY

MABEL CLARE WILLIAMS

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The experiments here reported were made for the purpose of determining the chief motives for visual illusions in the form of common objects. The research was planned primarily with reference to the analysis of some of the illusions found in the cylinder, but its scope gradually widened and led to the study of several other fundamental geometrical forms. The aim of the experiments was somewhat broader than simply to verify or disprove a particular hypothesis. The natural-history method of collecting and classifying data is the predominating feature of the plan. The aim has been to gather a large number of data systematically, especially in the form of judgments of persons who were not aware of the purpose of the study and who were also often unaware of the existence of the illusions, thus employing the so-called unconscious method as far as possible. This method of procedure is especially well adapted for the study of normal illusions, and in fact is really necessary, because normal illusions change when attention is directed to them and knowledge of the illusions leads to unconscious corrections. The naive judgments give a true measurement of the force of the illusion and new illusions are most readily discovered by this method. It is the puzzling residuals that appear unexpectedly and for some time baffle all efforts at explanation that in the end furnish the most reliable and convincing demonstration of fact.

The observers may be classified with reference to three general types: first, adults having knowledge of the illusions but making no conscious allowance for them; second,

adults without knowledge of the illusions; and third, children without knowledge of the illusions. The degree of knowledge of the subject possessed by each observer was ascertained as nearly as possible and the records interpreted with reference to this knowledge. In computing the results, the method of the average value has been employed. In many instances the median value might be more representative but for the sake of uniformity the average value has been used.

Each of the chief factors which modify the illusions has been varied in turn: the size, distance, form, and position of the object; the different classes of observers with regard to age, sex and intelligence; the different subjective conditions, such as the degree of knowledge, attention, and practice; and the different methods of judging.

The order of sequence adopted in the study of the different forms is due to two factors: first, the natural evolution of the research and second, the aim to avoid in each series and in all the series as a whole the effect of suggestion.

In every series where more than one judgment upon a point was secured from each observer the so-called double fatigue order was employed; that is, half the number of trials on each point were made in going over the series the first time, and, upon returning in the reverse order, the other half of the trials were made.

The horizontal line was selected as the best form in terms of which to state the illusions in the other forms studied. The line in this position is the simplest and most fundamental form and it seems to be freer from illusions than any other form.

The 114 mm. standard was adopted partly to relate this research more closely to other experiments in the same laboratory and partly for the reason that in other tests this size had been found to be very satisfactory. Only the most typical or fundamental forms were studied, it being



assumed that what is true for these would also be true in some measure for their derivatives.

The forms which were studied are shown in Figure 1, and they are named and described in the following list.

#### EXPLANATION OF FIGURE 1.

1. The square plate. 114 mm. x 114 mm.
2. The cube. 114 mm. x 114 mm. x 114 mm.
3. The cylinder. Length, 114 mm.; diameter 114 mm.
4. The sphere. Diameter 114 mm.
5. The triangle. Base 114 mm.; altitude 114 mm.
6. The pyramid. Base 114 mm.; altitude 114 mm.
7. The cone. Base 114 mm.; altitude 114 mm.
8. The disk. Diameter 114 mm.
9. The triangle and plate. Base 114 mm.; total length 228 mm.
10. The pyramid and cube. Base 114 mm. square; total length 228 mm.
11. The cone and cylinder. Diameter of base 114 mm.; total length 228 mm.
12. The circle. Diameter 114 mm.
13. The drawn square. 114 mm. x 114 mm.
14. The ellipse. Long axis 114 mm.; short axis 38 mm.
15. The drawn cylinder. Length 114 mm.; diameter 114 mm.
16. The line. Length 114 mm.

Forms 1, 2, 3, 4, 5, 6, 7, 9, 10, and 11 were made of tin and painted a lustreless black. Forms 8 and 14 were cut from dull black paper and pasted upon a sheet of light manilla cardboard one meter square. Form 1 was either cut from the paper or made of tin. Forms 12, 13, 15, and 16 were drawn upon a sheet of cardboard, the limiting lines being one millimeter wide. The end of the cylinder, Form 15, was represented by an ellipse the axes of which were in the ratio 3:1.



1



2



3



4



5



6



7



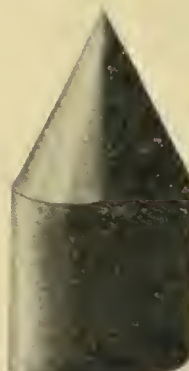
8



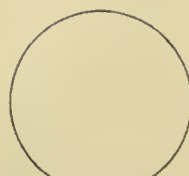
9



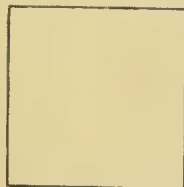
10



11



12



13



14



15



16

FIGURE 1





*Series I*

A preliminary study of the cylinder revealed a peculiar illusion which appears in the overestimation of its length. This will be called the illusion of cylinder length. The object of the experiments in Series I. was to collect measurements on this illusion in the length of the cylinder and the illusion of the vertical in the square. The experiments were made on school children in order to secure naive judgments and to discover how these illusions vary with age, sex and intelligence. They constituted one of nine systematic tests made upon one hundred and twelve pupils in the grammar school of Iowa City in the spring of 1900. The other eight tests were on different subjects, such as muscular fatigue, discriminative action, discrimination for the pitch of tones, hearing-ability, and keenness of vision. This combination of tests led the children to infer that their eyes were being tested and helped to keep them ignorant of the illusion. The experiments were made Saturday mornings. No results of this test were given out to the children.

Three measurements were made upon the cylinder and one upon the square, as follows:

Case 1. The vertical cylinder<sup>1</sup> at the level of the eyes: to select the one whose diameter and length are equal.

Case 2. The plate, at right angles to the line of vision: to select the one whose height and width are equal.

Case 3. The horizontal cylinder, at the level of the eyes: to select the one whose diameter and length are equal.

Case 4. The vertical cylinder, standing thirty degrees below the level of the eyes: to select the one whose diameter and length are equal.

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<sup>1</sup> The term "vertical cylinder" denotes a cylinder with the length in the vertical direction, and the term "horizontal cylinder" one whose length is in the horizontal direction.

The apparatus for Cases 1, 3, and 4 consisted of a series of fifteen cylinders (Fig. 1, Form 3) from which the observer was required to select the one in which the length appeared to be equal to the diameter. They were each 114 mm. in diameter but varied in length, by five-millimeter steps, from 69 mm. to 134 mm. For Case 2, the apparatus consisted of fifteen plates (Fig. 1, Form 1) each 114 mm. wide but varying in height, like the length of the cylinders, from 69 mm. to 134 mm. Both the cylinders and the plates were made of tin and were painted dull black. They were kept out of view in a case and the experimenter exhibited one at a time by placing it upon the center of a stand which was so adjusted, except in Case 4, that the middle of the form was at the level of the eyes of the observer. The stand was one foot square and was covered with gray cloth. The observer was at a distance of one and one-half meters.

The method of selection employed was the following: The cylinder whose length was equal to its diameter was presented first and the observer stated whether the length appeared to be equal to the diameter. If he said it was too long, the next shorter cylinder was presented and after that other short ones in order until the "equal" point had been passed and two consecutive cylinders had been pronounced too short; then the cylinders were presented in the opposite order until the "equal" point had been passed again and two consecutive cylinders had been pronounced too long. If, on the other hand, the observer said that the first cylinder presented was too short, the above order was reversed. Thus the judgments of the observers determined how many cylinders should be presented, and in what order, and the upper and the lower limits of the region of equality were determined twice for each observer. The procedure may be represented in the following scheme, in which the cylinders are denoted by the numbers indicating the length of the cylinder and the judgments upon

each by the letters. L denotes the judgment "too long"; E, "equal"; and S, "too short."

99	104	109	114	119	124
S	S	E	L	L	
	S	E	E	L	L

Here are two complete determinations; the first is 109 and the second is  $111\frac{1}{2}$ . The mean between these (110, neglecting fractions) is taken as the final record. Similar evaluations were made for other sequences of judgments. The same method was employed in the selection of the square.

The results are given in Tables *IA* and *IB*, which also contain the classification of the children according to age, sex, grade, school standing, and teacher's estimate of mental ability.

In Case 1 the average illusion for the boys is 17.5% and for the girls, 18.4%. This means that if the length of a vertical cylinder is to appear to be equal to the diameter of the same, the length must be about 18% shorter than the diameter. To the average boy the length of the vertical cylinder appears to be equal to the diameter when the length is 94 mm. and the diameter 114 mm., and boys vary from each other in this judgment by an average of only 4.4%. In this overestimation of the height of a cylinder, the illusion of the vertical and the illusion of cylinder length coöperate. But, when the cylinder is laid on its side, as in Case 3, they conflict and the illusion of the vertical causes an overestimation of the diameter while the illusion of cylinder length causes an overestimation of the length in a horizontal direction. If the two illusions were of equal strength, their effects would cancel each other, but the table shows that for these children the illusion of the vertical is on the average 1.8% stronger than the illusion of cylinder length.

In Case 2 the illusion of the vertical in the square was



TABLE I. (A). *Boys' Records.*

<i>Obs</i>	<i>Age</i>	<i>Gr</i>	<i>St</i>	<i>Ab</i>	<i>Case 1</i>		<i>Case 2</i>		<i>Case 3</i>		<i>Case 4</i>	
					<i>E</i>	<i>d</i>	<i>E</i>	<i>d</i>	<i>E</i>	<i>d</i>	<i>E</i>	<i>d</i>
2	16	8	B	B	93	4	102	3	122	0	90	4
4	13	8	D	B	101		100	1	97		109	3
6	15	8	D	C	94	3	98	1	125	4	95	4
8	14	8	D	C	95	4	100	5	115	1	88	7
10	16	8	E	E	93	4	109	0	103	1	102	3
12	14	8	E	D	95	4	104	0	120	2	83	7
14	13	8	A	A	95	2	109	0	112	2	98	4
16	15	8	A	A	103	4	108	1	119	0	100	2
18	14	8	B	C	93	1	100	3	113	7	90	4
20	13	8	C	B	85	1	99	0	127	3	87	3
22	12	8	C	B	93	4	105	1	115	2	99	3
24	14	8	A	B	89	3	104	0	109	0	99	
26	13	7	C	B	97	3	103	1	124	0	97	3
28	12	8	C	C	95	1	107	3	103	1	91	3
30	13	8	E	D	79	5	98	4	122	3	94	
32	12	8	C	B	104	3	110	1	109	0	107	3
34	13	8	D	C	85	1	99	3	122	3	84	3
38	15	8	C	C	84	3	99	0	122	0	85	4
40		8	D	C	84	0	108	1	118	1	89	
42	14	8	D	C	95	2	105	2	108	1	97	3
44	13	8	C	B	92	3	106	3	121	2	97	3
46	13	8	C	C	87	5	100	2	120	7	88	4
48	14	8	A	A	96	0	102	0	120	2	95	2
50	15	8	E	D	99	2	99	2	109	2	94	5
52	12	8	D	C	87	5	100	3	105	2	90	2
54	12	8	B	A	99	2	99	2	112	2	100	2
56	12	8	B	A	99	0	102	0	114	0	95	2
58	13	8	A	B	98	1	104	0	113	1	92	3
60	15	8	D	C	94	3	107	3	110	2	93	1
62	14	8	D	C	90	2	105	2	107	3	89	3
64	13	8	D	C	109	3	103	4	120	3	109	3
66	12	7	C	C	92	3	100	2	112	0	88	1
68	13	7	D	C	95	4	100	2	113	4	98	3
70	15	8	C	C	104	3	102	0	114	3	99	3
72	12	8	C	C	94	3	107	0	107	0	92	0
74	14	7	D	D	98	4	105	1	109	0	84	5
76	16	7	D	D	89	3	107	3	104	3	90	4
78	14	7	C	C	80	6	97	0	123	1	82	8
80	13	8	A	A	98	1	107	3	128	1	97	3
82	14	8	A	A	98	1	109	0	114	0	99	3
84	12	7	D	C	89	5	97	3	124	0	94	0
86	12	7	C	B	99	5	107	3	125	4	97	3

TABLE I. (A). Boys' Records. Continued.

Obs	Age	Gr	St	Ab	Case 1		Case 2		Case 3		Case 4	
					E	d	E	d	E	d	E	d
88	13	7	B	B	90	4	102	5	123	1	99	3
90	12	7	A	A	97	5	103	4	117	3	98	4
92	12	7	A	A	99	3	103	1	124	0	98	1
94	14	7	C	C	99	5	107	3	117	3	104	0
96	13	7	C	C	107	3	108	1	119	0	108	1
98	11	7	A	A	97	3	104	3	125	5	98	4
100	15	7	C	B	103	3	107	3	112	0	99	0
102	12	7	A	A	99	2	104	2	122	9	95	4
104	12	8	A	A	86	3	104	0	125	2	93	5
106	13	8	B	A	98	1	104	0	116	0	93	4
108	13	8	D	C	98	4	100	4	122	3	88	6
110	14	7	C	C	94	3	105	2	110	1	97	3
112	15	7	D	D	94	3	104	3	110	3	94	2
116	13	7	A	A	97	0	104	2	122	0	98	5
118	15	7	C	C	92	0	104	0	114	0	88	4
120	12	7	B	B	88	4	103	1	125	4	88	4
Average					94	3	103	2	116	2	95	3
Il					—20		—11		+2		—19	
% Il					—17.5		— 9.7		+1.8		—16.7	
% D					4.4		2.6		5.4		4.4	

Obs., the observers, by assigned numbers—boys even, and girls odd numbers.

Age, age at nearest birthday.

Gr., grade in the public school.

St., average standing according to the last monthly teacher's report.

Ab., mental ability as estimated by the teacher—five grades, A, B, C, D, and E in order, A being the highest.

E, the recorded estimate. This is the mean between two complete determinations, given in millimeters.

d, the mean variation, *i. e.*, the average deviation of the result for each trial from the average result for all the trials of the same observer, regardless of sign.

Il., the amount of the illusion, in millimeters.

% Il., the amount of the illusion, in percent of the standard distance (114 mm.)

% D., the average percent of deviation of the individual observers from the average for the group.

The minus sign signifies *below*, and the plus sign *above* the standard.

TABLE I. (B). *Girls' Records.*

<i>Obs</i>	<i>Age</i>	<i>Gr</i>	<i>St</i>	<i>Ab</i>	<i>Case 1</i>		<i>Case 2</i>		<i>Case 3</i>		<i>Case 4</i>	
					<i>E</i>	<i>d</i>	<i>E</i>	<i>d</i>	<i>E</i>	<i>d</i>	<i>E</i>	<i>d</i>
1	13	8	C	B	89	3	100	1	118	2	85	4
15	15	8	D	D	97		107		114		97	
21	14	8	D	D	97	0	105	2	107	3	96	4
23	13	8	C	B	100	2	107	3	120	2	95	4
25	15	8	C	C	97	3	104	3	112	3	87	5
27	13	8	C	B	85	2	97	0	117	3	89	5
29	13	8	D	C	100	3	108	1	109	2	100	2
31	13	8	D	D	82	8	105	2	115	4	89	7
33	14	8	D	D	84	3	88	1	119	3	95	4
35	15	8	A	B	93	1	100	4	112	5	93	4
37	14	8	E	E	93	4	105	2	109	0	97	3
39	14	8	E	D	84	5	104	3	94	5	74	3
41	14	8	C	C	77	3	97	5	103	2	81	0
43	16	8	D	D	83	9	98	4	117	3	80	6
47	14	8	C	C	95	2	104	0	107	0	89	3
51	13	8	C	C	97	3	105	2	112	0	95	2
53	14	8	A	A	89	3	105	2	118	2	92	0
55	15	8	C	C	98	1	104	3	118	1	94	3
57	13	8	D	C	94	5	104	3	109	3	89	3
59	13	8	B	B	104	3	109	0	118	1	102	3
61	13	8	B	C	95	4	102	3	118	1	93	4
63	12	8	A	A	95	7	105	2	119	0	94	3
65	14	8	A	A	88	4	103	1	114	0	86	2
67	14	7	D	E	109	3	109	3	118	1	107	0
69	11	7	E	E	102	0	99	0	120	1	94	5
71	14	7	D	C	95	5	102	3	118	1	90	2
73	12	7	C	B	89	3	108	1	123	4	95	4
75	11	7	D	D	96	5	99	3	109	0	90	4
77	13	7	D	D	104	5	110	2	105	2	88	5
79	11	7	A	A	85	2	99	0	115	2	105	4
81	14	7	B	B	85	6	108	1	123	1	94	5
83	16	7	D	D	92	3	104	3	118	3	92	3
85	13	7	C	C	95	2	107	3	124	0	87	8
87	12	7	A	A	99	3	104	3	114	3	94	3
89	11	7	A	A	99	2	108	1	124	3	97	3
91	13	7	B	B	98	1	103	1	122	5	98	1
93	12	7	D	D	90	4	99	3	120	2	89	5
95	13	7	B	B	85	4	99	0	124	0	95	5
99	12	7	C	C	89	5	98	1	119	3	94	5
101	13	7	C	C	90	4	107	3	120	3	94	3
103	11	6	D	C	97	3	105	2	124	3	98	5
107	14	7	D	E	92	4	103	4	114	3	90	4



TABLE I. (B). *Girls' Records.* Continued.

<i>Obs</i>	<i>Age</i>	<i>Gr</i>	<i>St</i>	<i>Ab</i>	<i>Case 1</i>		<i>Case 2</i>		<i>Case 3</i>		<i>Case 4</i>	
					<i>E</i>	<i>d</i>	<i>E</i>	<i>d</i>	<i>E</i>	<i>d</i>	<i>E</i>	<i>d</i>
109	14	7	C	C	88	1	104	3	121	1	90	1
113	12	7	A	A	90	6	102	3	113	1	91	5
115	12	7	A	A	90	6	97	3	122	0	90	4
117	16	8	B	B	94	3	104	3	112	0	94	0
119	13	7	A	A	99	5	105	2	122	0	104	0
121	11	7	B	B	105	4	99	2	119	2	94	5
123	14	8	D	C	93	4	104	2	125	4	98	5
125	13	7	C	C	95	6	104	2	114	0	103	1
129	13	7	D	C	93	1	107	3	109	0	98	4
131	13	7	B	B	85	4	104	0	114	0	94	5
135	15	7	B	B	94	0	108	1	112	0	95	1
139	13	8	B	B	100	4	97	3	132		102	
Average					93	4	103	2	116	2	93	3
II					—21		—11		+2		—21	
%II					—18.4		— 9.7		+1.8		—18.4	
%D					4.4		2.6		4.4		4.4	

measured. It amounts to 9.7% both for boys and for girls, and the children vary from one another in this judgment by an average of only 2.6%. It may be assumed for the present that the illusion of the vertical which enters into the overestimation of the height of the cylinder is the same as that for the square, the conditions being similar and the dimensions equal. If then, in the present method of comparison, the overestimation of the height of the cylinder is caused by two illusions only, the illusion of cylinder length amounts to 8.3%, which is found by subtracting 9.7 from 18. Again, if the illusion of the vertical in Case 3 is 9.7% for the diameter and the illusion of the vertical exceeds the illusion of cylinder length by 1.8%, then the latter amounts to 7.9%. There is thus less than 1% difference in the results for these two independent determinations of the illusion of cylinder length. The records demonstrate that the illusion of cylinder length exists independently of the illusion of

the vertical and is nearly equal to this familiar illusion, from which it has not to the writer's knowledge before been distinguished.

It was supposed that the cylinder would look longer when an end was in view, as it is shown in Fig. 1, Form 3, than when neither end could be seen, as in Cases 1 and 3. But the records for Case 4 reveal no appreciable difference. That is, the illusion of cylinder length is practically the same in the three fundamental positions in which the length of the cylinder may be seen.

There is no significant difference between the records of the boys and the records of the girls. The illusion is of about equal force, the mean variations are about equal, and the variation among the individuals is about the same for the boys and the girls.

In order to determine the variation of the illusion with age, these children may be taken as a group to represent children who are old enough to be intelligent observers and still are not mature mentally. They will be compared with adults in the next series.

To determine whether or not these illusions vary with mental ability, the records are classified, first, according to class standing (Table II) and, second, according to "mental ability" (Table III). The standing represents the averages from the last monthly reports. These are divided into five grades, represented by the letters in order, A being the highest. Mental ability was estimated by the teacher who knew the children best, and is equivalent to the dividing of the children into the five groups, "very bright," "bright," "average," "dull," "very dull."

The illusion of the vertical in the square does not vary with intelligence (See Case 2 in each table).

The illusion of cylinder length shows a tendency to be stronger for the less intelligent. This conclusion rests chiefly upon Case 3, for which both tables show that, in the conflict of the two illusions involved, the illusion of the

TABLE II.

<i>St</i>	<i>n</i>	<i>Case 1</i>			<i>Case 2</i>			<i>Case 3</i>			<i>Case 4</i>		
		% <i>Il</i>	<i>d</i>	% <i>D</i>	% <i>Il</i>	<i>d</i>	% <i>D</i>	% <i>Il</i>	<i>d</i>	% <i>D</i>	% <i>Il</i>	<i>d</i>	% <i>D</i>
A	23	—16.7	3	3.5	—8.7	2	1.8	+3.5	2	4.0	—15.8	3	2.6
B	17	—17.5	3	4.4	—9.6	1	2.0	+4.4	1	4.4	—16.7	3	2.6
C	32	—18.4	3	4.4	—9.6	2	2.6	+3.5	3	4.4	—17.5	3	5.2
D	33	—15.0	4	5.2	—9.6	3	2.6	0	2	5.2	—18.4	4	4.4
E	7	—19.3	3	5.2	—9.6	2	2.6	—2.6	2	7.0	—20.2	4	6.2

*n*, the number of children in each group.  
Other notation in this and the next table, same as in Table 1.

TABLE III.

<i>Ab</i>	<i>n</i>	<i>Case 1</i>			<i>Case 2</i>			<i>Case 3</i>			<i>Case 4</i>		
		% <i>Il</i>	<i>d</i>	% <i>D</i>	% <i>Il</i>	<i>d</i>	% <i>D</i>	% <i>Il</i>	<i>d</i>	% <i>D</i>	% <i>Il</i>	<i>d</i>	% <i>D</i>
A	23	—15.8	3	3.5	—8.7	2	2.0	+4.4	2	3.5	—15.8	3	3.0
B	27	—17.5	3	5.0	—8.7	2	3.0	+3.5	2	5.2	—15.8	3	3.5
C	41	—18.4	3	4.4	—9.6	2	2.6	+1.0	2	5.2	—18.4	3	3.5
D	16	—19.3	4	5.2	—10.5	3	2.6	—1.8	2	5.2	—21.9	5	4.0
E	5	—14.0	3	5.2	—8.0	2	2.6	—1.0	1	5.2	—14.0	3	4.4

vertical is stronger than the illusion of cylinder length for the very bright children, and the illusion of cylinder length is stronger than the illusion of the vertical for the very dull. This is also clearly corroborated in Cases 1 and 4 in both tables, for there is a variation in the combined illusion but the illusion of the vertical does not vary, as was shown in Case 2.

The figures in the %*D* columns show that the more intelligent children are in closer agreement with one another than the less intelligent. In other words, the illusion is more uniform for the bright than for the dull children.

The conclusions for this first series of experiments, upon the children, may be summarized as follows:

The illusion of the vertical for the square is about 10%.

The illusion of cylinder length is about 8%.

The illusion of cylinder length does not vary with the position of the cylinder.



Both the illusion of the vertical and the illusion of cylinder length are of approximately the same force for boys and girls.

The illusion of the vertical for the square does not vary with intelligence.

The illusion of cylinder length varies with intelligence.

The illusions are more constant for the bright than for the dull children.

### *Series II*

The purpose of this series was the further development of some of the problems presented in Series I. The observers, three women and eight men, were members of the University Summer Session of 1900. They were mature students, being teachers in the public schools of the state, but their knowledge of illusions was very vague and imperfect. In general the observers knew something of the illusion of the vertical but they knew nothing in regard to the other illusions which are under consideration.<sup>1</sup> They were given no suggestion of the nature of the problem, either before the experimenting began or during its progress.

The study of the illusions in the plate and the cylinder was continued by repeating the four cases of Series I. The plate was also compared with lines and the cylinder with lines and surfaces. The same plates and cylinders were used as in the previous series. Twelve cases were introduced into the series as follows:

Cases 1 to 4. The same as in Series I.

Case 5. The vertical cylinder: to select a plate whose width is equal to the diameter of the cylinder.

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<sup>1</sup> With regard to the illusion of the vertical, the observers belong mainly to type one, as defined on p. 38, but with regard to the illusion of cylinder length and other then unknown illusions, they belong to type two.

Case 6. The plate: to mark off a vertical distance equal to the height of the plate.

Case 7. The vertical cylinder: to select a plate whose height is equal to the length of the cylinder.

Case 8. The vertical cylinder: to mark off a vertical distance equal to the length of the cylinder.

Case 9. The vertical cylinder: to select a vertical line equal to the length of the cylinder.

Case 10. The plate: to select a vertical line equal to the height of the plate.

Case 11. Vertical cylinders: to select from a series of cylinders those whose heights are equal respectively to one-half, one, two, and four times the diameter.

Case 12. Rectangular plates: to select from a series of plates those whose heights are equal respectively to one-half, one, two, and four times the width.

In Cases 1 to 4 the same method was used as in Series I. The same method was also employed in Cases 5, 7, 9, and 10, except that the dimensions of two different objects instead of two dimensions of the same object were compared. For these four cases two large backgrounds, covered with seamless gray cloth, stood at right angles to each other; the standard form was placed in the center of one of these and the form to be compared with it in the center of the other. For Cases 9 and 10, a series of fifteen wires, 2.5 mm. in diameter and varying in length from 69 to 134 mm., served as lines. A small hole, scarcely visible at the observer's distance, was drilled near one end and the wires were hung upon a projecting pin as needed.

In Cases 6 and 8 a method of production was used. Two tables were placed at right angles to each other and had erected upon them backgrounds of the same dimensions as the tops of the tables. Both tables and backgrounds were covered with gray cloth. The standard form was placed upon one table against the middle of the background and the observer indicated upon the other background with a

pointer one meter long, a distance above the table which he judged to be equal to the height of the standard form.

The apparatus for Case 11 was a series of twenty-three wooden cylinders painted black, each having a diameter of 114 mm. The lengths were 36, 40, 45, 52, 57, 64, 71, 80, 90, 101, 114, 128, 143, 163, 182, 223, 228, 256, 288, 324, 365, 407, 456 mm. These cylinders were placed upon a table in the order of size and about 50 mm. apart, and the observers simply pointed to the cylinders they selected.

In Case 12 rectangular black cardboard plates, 114 mm. wide and corresponding in height to the lengths of the wooden cylinders, were used. They were fastened invisibly to a long strip of gray cloth, 90 cm. wide, which was tacked upon the wall.

In addition to the measurements upon the forms 114 mm. in length, measurements were also made upon the forms 104 mm. and 124 mm. in length for Cases 6 to 10 inclusive. The purpose of the tests upon these forms was to secure a check upon the measurements on the regular 114 mm. standards and also to eliminate any suggestion which might rest upon the employment of forms that have equal dimensions.

All the forms were on a level with the observer's eyes, unless otherwise noted, and one and one-half meters away. Four determinations were made in each of the first ten cases and only one determination in each of the last two cases.

The averages of all the estimates of the observers, the average individual mean variations, and the percentage of illusion for the various cases are presented in Table IV. In Table V the individual estimates are given for the 114 mm. square and 114 mm. cylinder. This table is introduced to show the extent of agreement of the different observers with one another.



TABLE IV.

Case	Standard 104 mm.			Standard 114 mm.			Standard 124 mm.			E	% Il
	E	d	% Il	E	d	% Il	E	d	% Il		
1				100	3	—12					
2				107	2	— 6					
3				109	2	— 3.5					
4				97	3	—15					
5				123	3	+ 8					
6	108	3	+ 3.8	116	3	+ 1.8	126	3	+ 1.6		
7	121	3	+16	130	2	+14	135	1	+ 9		
8	116	3	+12	127	3	+11	136	4	+ 9		
9	114	3	+10	125	3	+10	134	1	+ 8		
10	106	2	+ 2	115	2	+ 0.9	125	2	+ 0.8		
	Standard 57 mm.			Standard 114 mm.			Standard 228 mm.			Standard 456 mm.	
	E	d	% Il	E	d	% Il	E	d	% Il	E	% Il
11	54		— 5	93		—18	204		—11	366	—20
12	55		— 3	112		— 1.8	211		— 7	383	—16

The notation is the same as in Table I.

TABLE V.

Case	1	2	3	4	5	6	7	8	9	10	11	12
Obs	E	d	E	d	E	d	E	d	E	d	E	E
2	99	3	107	3	114	0	102	3	115	4	113	0
4	103	2	105	2	108	1	96	3	119	0	120	3
1	103	4	107	0	109	0	94	0	131	1	125	5
6	105	3	107	3	105	2	103	4	120	6	112	5
8	101	2	110	2	110	3	96	4	125	2	117	2
10	94	5	109	0	114	1	99	3	117	1	108	3
5	97	3	107	0	108	1	92	3	121	3	115	4
3	94	0	102	0	110	2	89	3	123	1	127	5
12	102	3	106	3	98	4	91	2	134	5	112	4
14	102	5	109	2	109	3	107	3	122	3	109	3
16	97	2	111	2	110	4	94	1	125	1	121	3
Ave	100	3	107	2	109	2	97	3	123	3	116	3
Il	—14		— 7		— 5		—17		+ 9		+ 2	
% Il	—12		— 6		— 3.5		—15		+ 8		+ 1.8	
% D	2.6		1.8		2.6		4		4		4.4	

The notation is the same as in Table I. In the first column the even numbers refer to the men and the odd numbers to the women. The individual records are presented in the order in which they were made.

The plate was studied in Cases 2, 6, 10, and 12. From Case 2 there is obtained a measurement of the illusion of the vertical. This is shown in the selection of a plate whose height is 6% less than the width; that is, a plate 107 mm. tall and 114 mm. wide was judged to be square. The result for the same experiment upon the school children was -9.7% (Series I, Case 2). In Case 6, where the height of the plate was marked off with a pointer, and in Case 10, where the line equal to the height of the plate was selected, the illusion of the vertical appears in both the standard and compared forms and is therefore fairly eliminated. In Case 6 there is a residual of +1.8% and in Case 10 of +0.9%. These residuals are due to other illusions which will be discussed in a subsequent series. For the 114 mm. standard in Case 12, the illusion of the vertical is smaller than usual. This is probably due to the use of a long series of parallelograms, and also to the fact that the observers reacted against the illusion of the vertical.

The tests upon standards 57, 228, and 456 of Case 12 are interesting, since they represent the balancing of two illusions. With these forms there is in addition to the illusion of the vertical, the illusion of length, according to which the longer dimension of a parallelogram is overestimated.<sup>1</sup>

When the form is longer in the horizontal dimension, as with standard 57, the illusion of the vertical and the illusion of length conflict, the former tending to cause the selection of a plate whose height is too small and the latter one whose height is too great. The result is -3%; that is, the illusion of the vertical is stronger than the illusion of length by 3% in this instance. When the height of the form is two and four times the width (Standards 228 and 456 respectively), the illusion of the vertical coöperates with the illusion of length, the effect of each being to cause

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<sup>1</sup> See SEASHORE and WILLIAMS, *An Illusion of Length*, Psychol. Rev., 1900, VII., 592. Reprinted in this volume, p. 29.

the selection of a plate which is too short vertically. The forms actually selected were too short by 7% and 16% respectively. The sum of the two illusions apparently increases with the length of the form. In these experiments there is no way of determining whether this variation is in one or both of the illusions.

The illusion for the vertical cylinder is 12% for Case 1; that is, the length and diameter of a cylinder appear equal when the length is 12% less than the diameter. The child observers selected a cylinder which was 18% too short. (Series I, Case 1). These amounts represent the coöperation of the illusion of the vertical and the illusion of cylinder length. The former illusion is 6% for the plate and it may be assumed that it is the same for the length of the cylinder. The illusion of cylinder length then amounts to 6%.

It was stated in the discussion of Series I that the illusion of the vertical and the illusion of cylinder length conflict when the cylinder is laid upon its side as in Case 3. The result for this case is  $-3.5\%$ ; that is, the illusion of cylinder length outweighs by 3.5% the illusion of the vertical. If the illusion of the vertical is 6%, then the illusion of cylinder length is 6% plus 3.5% or 9.5%. In order to determine the variation of the illusion of cylinder length with the age of the observers, these two statements of the illusion for adults, 6% and 9.5%, should be compared with the corresponding statements of the illusion for children, 8.3% and 7.9%, as shown in Series I.

In both Case 4 and Case 1, the length of the vertical cylinder was compared with the diameter but the observers occupied different positions with reference to the cylinder. In Case 4 they looked down upon it at an angle of thirty degrees. The illusion is 3% greater for Case 4 than for Case 1. This is probably due to an underestimation of the diameter on account of the Müller-Lyer effect. The illusions for the cylinder are of greater force in Case 11,



standard 114, than in Case 4. With the other three standard cylinders of Case 11, the illusion of length, the illusion of the vertical, and the illusion of cylinder length enter. In general these illusions, taken together, seem to increase with the increase in the length of the cylinder.

In the discussion of the cylinder up to this point, only the ratio of the length and diameter has been considered. In Case 5 the diameter of the cylinder was compared with plates with the result that a plate 8% wider than the horizontal diameter of the cylinder was selected as equal to it. This means that there is an overestimation in the diameter (width) of the cylinder which does not appear in the width of the plate. For the present this illusion will be called the illusion of cylinder diameter but it will be explained and renamed in the following series of experiments. Now, it is evident that if the diameter of the cylinder is overestimated, the results in those cases in which the length and diameter of the cylinder are compared do not represent the full force of the illusions involved. For instance, the illusion of length in the cylinder in Case 1 is of sufficient strength to exceed by 6% the illusion of cylinder diameter, which is 8%. The illusion of cylinder diameter would cause too long a cylinder to be selected but, eliminating the illusion of the vertical, a cylinder 6% too short is selected. The illusion in the length of the cylinder, exclusive of the illusion of the vertical, then, really amounts to 14% in Case 1. It will be observed that the illusion of cylinder length is only a part of the illusion in the length of the cylinder. A further analysis of this fact will be given in the next series. In Case 7 the height of the plate was compared with the length of the cylinder. By this method the illusion in the length of the cylinder is found to be 14%. The results obtained by these two radically different methods are the same. Two more independent statements of the illusion in the length of the cylinder are obtained from Cases 8 and 9. These are 11% and 10%

respectively for the two cases. These figures would be larger if the methods used did not involve so many complications. In Case 8 the presence of the base line, and in Case 9 the presence, in the wire, of a new illusion to be discussed later, reduces the force of the illusions.

The illusion of cylinder length may be estimated to be about 8%, and, taking all the above data into consideration, the total illusion in the length of the cylinder, exclusive of the illusion of the vertical, is 14%. There is therefore a residual of about 6% in the length of the cylinder.

All the essential features found in the experiments with the 114 standard (the form in which the two dimensions are equal) are confirmed by the results for the experiments upon the two forms in which the dimensions were not equal (Standards 104 and 124 of Cases 6 to 10 inclusive). The difference in the proportions of the two dimensions of the form has however a marked effect, as may be seen in Table IV.

The general conclusions drawn from the results of this series of experiments, under the conditions described, may be summarized as follows:

The illusion of the vertical in the square is about 6% for this type of observers.

The illusion of the vertical in the square is not so strong for the adults as for the children. (Series II, Case 2; Series I, Case 2).

The illusion of cylinder length is apparently as strong for adults as for children.

A new illusion appears in the overestimation of the height of the plate. (Cases 6 and 10).

The total effect of the illusion of the vertical and the illusion of length increases with the length of the parallelogram. (Case 12).

The total effect of the illusion of the vertical, the illusion of cylinder length, and the illusion of length increases with the length of the cylinder. (Case 11).

The illusion of cylinder length, as determined in two different ways, is found to be stronger than the illusion of the vertical.

The diameter of the cylinder is overestimated by about 8%. (Case 5).

There is a new illusion in the length of the cylinder which appears in an overestimation of about 6%.

The difference in the proportions of the two dimensions of an object influences the perception of the magnitude of one dimension. (Standards 104 and 124 of Cases 6 to 10 inclusive).

### *Series III*

The experiments in Series III were made in the fall of 1900. The purpose of this series was, first, to establish further the illusions in the cylinder and square, which were brought out in the previous series, by studying them under more varied conditions and with different observers; and, second, to obtain data upon two new forms, the cube and drawn cylinder (Fig. 1, Forms 2 and 15).

The standard cube and cylinder were placed upon a small support in the center of a background of light manilla cardboard 1 meter square. The plate was cut from black paper and pasted at the center of a sheet of cardboard and the drawn cylinder was outlined at the center of another sheet. These backgrounds were of the same color and size as the one upon which the standard forms were placed.

The forms used for comparison with the standard forms were lines and plates (Fig. 1, Forms 16 and 1). A series of twelve black lines, 1 mm. in width and varying in length by 5 mm. steps, from 94 mm. to 149 mm., were drawn upon each of four manilla cardboard sheets, 1 meter square. The lines were drawn in no definite order with regard to length, but they were 50 mm. apart and parallel.



The arrangement of the lines was different in the four series.

There were four series of twelve rectangular plates each. These plates were all 114 mm. wide, but the lengths varied by 5 mm. increments from 94 mm. to 149 mm. in each series. They were fastened upon a background 2 meters square, covered with the light manilla cardboard. The four series were placed in as many rows; in the first the smaller plates were in the center and the larger ones at the ends and they varied in the vertical dimension; in the second row the smaller plates were also in the central part but the horizontal dimension varied; in the third row the larger plates occupied the central position and their vertical dimensions varied; and in the fourth row the larger plates were in the center and their horizontal dimensions varied. The plates were about 45 mm. apart in the row and the rows about 30 cm. apart. The height of this large background was adjustable and it was so arranged that the particular row from which the plate was to be selected was on a level with the eyes of the observer. By having several series of plates and lines the tendency of the observers to select from memory was obviated.

The background for the standard forms and that for the compared forms were placed at right angles to each other. The observer was seated upon a revolving office stool at a distance of 1 meter from the objects observed and was required to turn upon the stool so that the whole body moved through an angle of ninety degrees. The standard form was first placed before the observer, then a series of lines or plates with which the standard was to be compared was shown. The observer indicated with a long pointer the compared form which he selected.

Measurements were also made upon the cube, the cylinder, and the plate when these forms were so placed that the observer looked down upon them at an angle of thirty degrees. In the case of the cube and cylinder this afforded a view of the top. The forms with which the standard

TABLE VI.

<i>Case</i>	<i>Dimension of Standard Form Measured</i>	<i>Compared Form</i>	<i>Direction</i>	<i>No. in Fig. 1</i>
1	Height of cube	Lines	Vertical	2
2	Width of cube	Lines	Horizontal	2
3	Length of cylinder	Lines	Vertical	3
4	Diameter of cylinder	Lines	Horizontal	3
5	Height of cylinder	Plates	Vertical	3
6	Diameter of cylinder	Plates	Horizontal	3
7	Height of cube	Plates	Vertical	2
8	Width of cube	Plates	Horizontal	2
9	Length of cylinder	Plates	Horizontal	3
10	Diameter of cylinder	Plates	Vertical	3
11	Length of cylinder	Lines	Horizontal	3
12	Diameter of cylinder	Lines	Vertical	3
13	Length of drawn cylinder	Lines	Vertical	15
14	Diameter of drawn cylinder	Lines	Horizontal	15
15	Length of drawn cylinder	Plates	Vertical	15
16	Diameter of drawn cylinder	Plates	Horizontal	15
17	Length of drawn cylinder	Lines	Horizontal	15
18	Diameter of drawn cylinder	Lines	Vertical	15
19	Length of drawn cylinder	Plates	Horizontal	15
20	Diameter of drawn cylinder	Plates	Vertical	15
21	Length of cylinder*	Lines	Vertical	3
22	Length of cylinder*	Plates	Vertical	3
23	Height of cube*	Lines	Vertical	2
24	Height of cube*	Plates	Vertical	2
25	Height of plate	Plates	Vertical	1
26	Height of plate	Lines	Vertical	1
27	Height of plate*	Plates	Vertical	1
28	Height of plate*	Lines	Vertical	1
29	Diameter of cylinder*	Lines	Horizontal	3
30	Diameter of cylinder*	Plates	Horizontal	3

\* The observer looked down upon these forms at an angle of 30 degrees.

forms were compared when in this position were on a level with the observer's eyes.

Except as just noted, only one side of the cube was visible and neither end of the cylinder. Eight trials were made on each case; the experiment lasted one hour a day for each observer and four days were required for some of the observers. The order of sequence of the cases was deter-

TABLE VII.

<i>Case</i>	<i>A</i>	<i>VI</i>	<i>C-L</i>	<i>M-L</i>	<i>% Il</i>
1*	+	+	0	0	+12
2*	+	+	0	0	+14
3*	+	+	+	0	+15.7
4*	+	+	0	0	+12
5†	0	+	+	0	+ 9
6	0	+	0	0	+ 6.1
7†	0	+	0	0	+ 3.7
8	0	+	0	0	+ 5.2
9	0	+	+	0	+12.3
10†	0	+	0	0	0
11*	+	+	+	0	+18.5
12*	+	+	0	0	+10.4
13*	+	+	+	0	+17.5
14*	+	+	0	0	+12.1
15†	0	+	+	0	+11.7
16	0	+	0	0	+ 5.2
17*	+	+	+	0	+20
18*	+	+	0	0	+ 9.5
19	0	+	+	0	+16.7
20†	0	+	0	0	+ 1.1
21*	+	+	+	0	+14.8
22†	0	+	+	0	+ 9
23*	+	+	0	0	+11.3
24†	0	+	0	0	+ 3
25†	0	0	0	0	0
26*	+	0	0	0	+ 7
27†	0	0	0	0	0
28*	+	0	0	0	+ 6
29*	+	+	0	—	+13
30	0	+	0	—	+ 3.5

*A*, area illusion.

*VI*, volume illusion.

*C-L*, cylinder-length illusion.

*M-L*, Müller-Lyer illusion.

*% Il*, percentage of illusion.

\*In these cases an allowance of 1% has been made to eliminate the error due to the series of lines.

†An allowance of 5% has been made for the plates for a similar reason.



TABLE VIII.

Case	<i>Obs 2</i> (4 trials)		<i>Obs 4</i> (8 trials)		<i>Obs 6</i> (8 trials)		<i>Obs 8</i> (8 trials)		<i>Obs 10</i> (8 trials)		<i>Obs 12</i> (6 trials)		<i>Obs 14</i> (8 trials)		<i>Obs 16</i> (2 trials)	
	<i>E</i>	<i>d</i>	<i>E</i>	<i>d</i>	<i>E</i>	<i>d</i>	<i>E</i>	<i>d</i>	<i>E</i>	<i>d</i>	<i>E</i>	<i>d</i>	<i>E</i>	<i>d</i>	<i>E</i>	<i>d</i>
1	130	2	141	8	133	4	118	4	117	4	134	3	119	5	129	5
2	133	3	130	8	136	3	125	4	115	2	137	6	134	6	129	5
3	132	3	144	3	137	6	120	5	123	6	140	2	128	5	132	3
4	133	2	128	8	133	4	122	4	118	3	132	5	135	7	122	3
5	124	0	138	5	124	1	122	3	123	3	134	5	133	5	132	3
6	118	2	127	5	119	5	115	2	121	4	119	3	123	4	117	8
7	119	0	128	6	116	3	117	2	119	2	126	4	130	3	129	0
8	115	2	127	6	123	2	117	3	118	3	122	2	127	4	117	8
9	119	0	129	6	131	2	123	2	129	4	131	5	137	3	124	0
10	118	2	130	4	113	1	117	3	120	3	126	2	128	4	117	3
11	135	2	137	3	140	4	128	3	124	5	142	6	138	8	129	10
12	128	2	137	8	130	4	119	5	118	5	133	4	121	5	124	0
13	133	2	135	9	141	5	131	3	122	3	137	3	131	6	142	3
14	137	4	136	7	134	5	128	4	117	3	132	4	129	6	124	0
15	130	4	134	5	130	5	131	2	130	2	138	4	133	8	139	0
16	120	2	127	4	118	3	118	2	122	2	116	2	127	6	122	3
17	138	4	133	7	138	7	130	3	126	4	140	5	145	3	144	0
18	130	2	137	6	131	2	123	2	113	2	127	5	118	3	129	5
19	124	0	130	7	133	2	126	2	135	3	129	3	136	3	144	0
20	120	2	128	5	115	3	122	3	119	3	120	2	121	3	119	0
21	132	3	139	6	138	3	116	4	120	4	142	4	126	6	134	0
22	124	0	135	6	129	3	117	2	126	2	137	2	132	4	139	0
23	129	0	142	6	126	4	113	3	117	4	138	3	122	6	124	0
24	119	3	132	4	117	3	114	3	121	2	133	4	128	4	124	5
25	117	3	117	6	113	1	120	1	117	2	123	4	123	2	127	3
26	129	0	125	6	125	3	118	3	115	3	131	4	119	4	124	0
27	117	5	114	6	111	2	115	2	115	4	127	5	124	3	127	3
28	124	3	120	8	122	4	115	7	112	4	134	0	117	3	122	3
29	129	5	132	5	137	2	118	3	115	3	137	8	129	2	119	0
30	117	3	121	4	113	2	110	2	118	3	124	5	123	4	112	3

mined partly to avoid disturbing influences in the serial order and partly by the convenience in manipulating the apparatus. The double fatigue order was observed. Vertical distances were compared with vertical and horizontal with horizontal.

The various cases introduced in Series III are described in Table VI. The dimension of the form measured, the form with which it was compared, and the direction of the

TABLE VIII. Continued.

<i>Obs 18</i> (2 tr'ls)		<i>Obs 20</i> (2 tr'ls)		<i>Obs 22</i> (2 tr'ls)		<i>Obs 1</i> (1 tr'l)		<i>Ave</i>		<i>Il</i>	% <i>Il</i>	% <i>D</i>	<i>Case</i>
<i>E</i>	<i>d</i>	<i>E</i>	<i>d</i>	<i>E</i>	<i>d</i>	<i>E</i>		<i>E</i>	<i>d</i>				
122	3	134	5	132	3	134		129	4	+15	13.1	5.2	1
129	5	129	5	139	5	139		131	5	+17	15.0	4.4	2
134	0	134	5	137	8	134		133	4	+19	16.7	4.4	3
127	3	124	0	139	10	134		129	4	+15	13.1	4.4	4
122	3	124	5	139	5	144		130	3	+16	14.0	6.1	5
117	3	104	0	134	5	134		121	4	+ 7	6.1	5.2	6
122	3	124	5	124	5	139		124	3	+10	8.7	4.4	7
119	0	114	0	124	0	129		120	3	+ 6	5.2	3.5	8
119	5	124	0	129	5	139		128	3	+14	12.3	4.4	9
114	5	114	0	119	10	129		120	3	+ 6	5.2	4.4	10
127	3	139	5	144	5	144		136	5	+22	19.3	5.2	11
122	3	129	5	132	3	129		127	4	+13	11.4	4.4	12
137	8	139	5	139	10	129		135	5	+21	18.4	3.5	13
127	3	124	0	137	3	124		129	4	+15	13.1	4.4	14
129	0	129	0	139	0	139		133	3	+19	16.7	3.5	15
117	3	104	0	124	0	119		120	2	+ 6	5.2	4.4	16
137	3	139	5	149	0	139		138	4	+24	21.0	4.4	17
124	10	119	0	137	8	119		126	4	+12	10.5	5.2	18
134	0	132	3	134	0	139		133	2	+19	16.7	3.5	19
124	5	112	3	129	10	129		121	4	+ 7	6.1	3.5	20
127	8	137	3	129	0	139		132	4	+18	15.8	6.1	21
124	5	124	5	132	3	144		130	3	+16	14.0	5.2	22
124	0	137	3	132	3	129		128	3	+14	12.3	6.1	23
117	3	119	5	119	0	129		123	3	+ 9	8.0	5.2	24
114	0	117	3	119	0	129		120	2	+ 6	5.2	3.5	25
119	5	132	3	129	5	114		123	3	+ 9	8.0	4.4	26
124	5	114	0	119	0	124		119	3	+ 5	4.4	4.4	27
124	0	132	3	132	3	114		122	3	+ 8	7.0	5.2	28
134	0	124	0	132	3	129		130	3	+16	14.0	5.2	29
127	3	112	3	124	0	119		118	3	+ 4	3.5	4.4	30

Notation same as in Table I.

linear dimensions compared are given. The averages of the individual observers are found in Table VIII. This table reads across from left to right for each case and from the top down for each observer. An analysis of the illusions involved in the forms is found in Table VII, which is placed opposite Table VI for convenience in comparison. In each case the appropriate sign is given for each illusion

that enters; a plus sign signifies that the compared dimension selected should be larger than the standard dimension, and a minus sign signifies that the compared dimension selected should be smaller than the standard dimension. The same illusion may have a plus or a minus sign, depending upon the method of comparison. A plus sign in the II. column indicates that the compared dimension was selected larger than the standard dimension.

The rather large mean variations in these tables are due to the method employed, a discussion of which will be given in connection with a later series. The very large variation given under %D is accounted for by the various degrees of knowledge and preparation represented by the observers, and by the fact that the force of the same illusion often varies greatly with different individuals.

The conclusions drawn from Series III are based upon the results obtained from twelve representative observers. They will be mentioned individually, as they represent different degrees of training and knowledge of the subject of visual illusions. Observer 2 had an extensive knowledge of the subject and was an observer of long experience; he made a strong effort not to let himself be influenced by his theories. Observers 4 to 14, even numbers inclusive, represent a class of second year students in psychology. They had practically equal knowledge of the illusions and were all about equally experienced as observers. Observer 16 was an instructor in philosophy. Observer 18 was an instructor in mathematics trained in estimating distances by the eye but he had no definite knowledge concerning any illusions except the illusion of the vertical. Observer 20, an instructor in the department of civil engineering, had never studied illusions but was a skilled draughtsman. Observer 22 was a first year student in civil engineering; he knew nothing whatever about illusions and his judgments may be regarded as more naive than the judgments of any of the other observers. Observer 1 may be classed with



observers 4 to 14 mentioned above. To none except Observer 2 was the information given that the test was upon visual illusions and, since the attention was not called to this fact, they as a rule made no conscious allowance for the illusions. With regard to the illusion of the vertical, all but Observer 20 belong to the first type of observers; but with reference to the other illusions, the Müller-Lyer illusion excepted, all but Observer 2 belong to the second type.

In the interpretation of the records, allowance must be made for an error which occurs as the result of placing the lines and plates in a series. When a line is grouped with several other lines, as in the charts described on p. 60, there is a tendency to see it shorter than it really is, regardless of direction. The amount of this underestimation has been carefully measured and will come up for discussion in the proper place under Series VII. It is necessary at this point only to note that the error in the series of lines amounts to 1%. This must be deducted from the results in all those cases in which the standard forms were compared with lines. The error in the series of plates is similar in nature but is of greater force. Its measurement is obtained from Case 25 of the present series. (See Table VIII). Here a plate is selected from the series of plates which appears equal to the height of the standard plate. Now if each form stood alone in such a comparison, there should be no illusion, but as the standard plate stands alone and the other plate is one of a series of plates, there enters an illusion of about 5%. It will be necessary to deduct this amount from all those cases in which the standard forms were compared with the height of the plates. This 5% may not be the same for all the cases but, for the present purpose, it will be necessary to assume that the error due to the series of plates is constant. These corrections have been made in Table VII by subtracting 1% for lines and 5% for plates from the original results as given in Table

TABLE IX.

<i>Dimension of Standard Form Measured</i>	<i>Compared with Lines</i>				<i>Compared with Plates</i>			
	<i>Vertical % Il</i>	<i>Horizontal Case</i>	<i>Vertical % Il</i>	<i>Horizontal C'e</i>	<i>Vertical % Il</i>	<i>Horizontal C'e</i>	<i>Vertical % Il</i>	<i>Horizontal C'e</i>
Height of plate	7.0	26*			0	25†		
H'ght of plate, 30° down	6.0	28*			0	27†		
Cube	12.0	1*	14.0	2*	3.7	7†	5.2	8
Cube, 30° down	11.3	23*			3.0	24†		
Length of cylinder	15.7	3*	18.3	11*	9.0	5†	12.3	9
L'gth of cyl'r, 30° down	14.8	21*			9.0	22†		
Length of drawn cyl'r	17.4	13*	20.0	17*	11.7	15†	16.7	19
Diameter of cylinder	10.4	12*	12.0	4*	0	10†	6.1	6
Diam. of cyl'r, 30° down			13.0	29*			3.5	30
Diam. of drawn cylinder	9.5	18*	12.0	14*	1.1	20†	5.2	16

\*In these cases an allowance of 1% was made, to eliminate the error due to the series of lines.

†An allowance of 5% was made for the plates, for a similar reason.

VIII. From the data available there is no definite means of ascertaining whether or not the width of the plate is affected when the plate is one of a series, therefore no allowance will be made in those cases where the width of the plate was compared with the standard form. In Table IX, which is a summary of Table VIII, the appropriate deduction of 1% for the lines and 5% for the plates has also been made, and it is upon this table that the discussion of this series is based.

The results of the measurements upon the standard plate, cube, cylinder, and drawn cylinder, when these forms are on a level with the observer's eyes, will be discussed first; then the corresponding measurements upon the forms when thirty degrees below the level of the eyes will be considered.

The plate will be examined first. In case 26 its height was compared with a vertical line. The result is +7%; that is, a line must be 7% higher than the plate if the two forms are to appear equal in height. In other words, if the line and plate are actually equal in height, the plate ap-



pears to the observers to be 7% higher than the line. The question—Why is this? at once arises. The effect of the illusion of the vertical is practically eliminated, for it appears in both forms and so does not change the relative sizes of the line and plate to any great extent. The forms differ from each other only in one respect; the plate has area while the line is practically without area. The difference between these forms must in some way be due to this fact. The illusion of 7% in the plate is an illusion due to area; it is on account of the width of the plate that its height appears greater than the height of the line. Hence we name this the area illusion. It is more thoroughly established in the following series and its application to the series now under discussion is therefore justified.

The cube (Fig. 1, Form 2) was introduced for the first time in Series III. In Case 1 its height was compared with a vertical line and in Case 2 its width with a horizontal line. There resulted an overestimation of 12% and 14% respectively for the two cases. Part of this is evidently due to the area illusion brought out in connection with the plate. When the cube is compared with the plate, as in Cases 7 and 8, the area illusion appears in both forms and is therefore to be disregarded. Yet in Case 7, where the height of the cube is compared with the height of the plate, there is required a plate 3.7% higher than the cube to appear equal to it; and when the width of the cube is compared with the width of the plate (Case 8), a plate 5% wider is required. The cube was so placed that only one face of it was visible to the observer; it thus had the same appearance to him as the plate, although he was fully aware that it was a cube. Now, the cube differs from the plate only in that it has the three dimensions of space; therefore any difference between the results for the two forms must be regarded as due to this fact. In other words, if the cube is judged to be larger than the plate it is because it has volume. The percentages obtained in Cases 7 and 8 and



the residuals in Cases 1 and 2 after the area illusion has been eliminated are statements of an illusion due to volume; hence we name it the volume illusion.

A line has relatively neither volume nor area. A vertical line appears equal to the height of the cube only when it is 12% taller than the cube (Case 1) and a horizontal line appears equal to the width of the cube only when it is 14% longer than the width of the cube (Case 2). These are statements of the area and volume illusions combined. The plate, which has area, appears equal in height to the height of the cube when it is 3.7% taller than the cube (Case 7) and it appears equal to the cube in width only when it is 5.2% wider than the cube (Case 8). These percentages are statements of the volume illusion in the cube. The area illusion for the height of the cube is 12% minus 3.7% or 8.3% (Case 1 minus Case 7), and for the width of the cube it is 14% minus 5.2% or 8.8% (Case 2 minus Case 8); that is, the area illusion is the difference between the results for the comparisons of the cube with lines and with plates.

Turning now to the cylinder (Fig. 1, Form 3), it is seen that when a vertical line is selected which appears equal to the length of the vertical cylinder, there is an illusion of 15.7% (Case 3); and when a horizontal line is selected which appears equal to the length of the horizontal cylinder, there is an illusion of 18.3%. Further, when the height of the plate is compared with the length of the vertical cylinder (Case 5), and the width of the plate with the length of the horizontal cylinder (Case 9), the illusions are reduced to 9% and 12.3% respectively for the two cases. That is, when the forms compared are vertical, a line must be 15.7% higher than the length of the cylinder in order to appear equal to it, whereas a plate need be but 9% higher; and when the forms compared are horizontal, a line must be 18.3% longer than the length of the cylinder in order to appear equal to it, whereas a plate need be but 12.3% wider.

This difference between the line and plate is about 6% for the vertical position and the same for the horizontal position of the forms. (Compare Case 3 with 5 and Case 11 with 9). This 6% is the area illusion in the length of the cylinder, for the motive of the area illusion found in the plate is present in the cylinder. The motive for the volume illusion found in the cube is also present in the cylinder, but in these measurements its force for the length of the cylinder is not determined directly, since it appears in combination with the illusion of cylinder length, for which no determination was made in the present series. However, in Series II this illusion was found to amount to 8% and it is probably about the same in the present series. The area illusion in the length of the cylinder is 6%; and in Case 3, where the vertical line is compared with the vertical cylinder, three illusions enter, exclusive of the illusion of the vertical, namely, the illusion of cylinder length, the area illusion and the volume illusion. The combined force of these illusions is 15.7%. Subtracting 8% for the illusion of cylinder length and 6% for the area illusion, there is a residual of 1.7% which is the volume illusion in the length of the vertical cylinder. Similarly, it is 4.3% for the length of the horizontal cylinder, when the length of the cylinder is compared with a horizontal line (Case 11). When the plate is compared with the length of the cylinder, only the illusion of cylinder length and the volume illusion enter (Cases 5 and 9). If the illusion of cylinder length is 8%, the volume illusion for the length of the vertical cylinder is 1% (Case 5), and for the length of the horizontal cylinder it is 4.3% (Case 9). Of these four statements of the volume illusion for the length of the cylinder, the two upon the length of the horizontal cylinder agree with each other and those upon the length of the vertical cylinder agree with each other, but the two determinations of the illusion for the vertical cylinder are smaller than those for the horizontal cylinder. This is due



to the fact that not all of the illusion of the vertical is excluded in the tests upon the vertical cylinder, a fact which will be fully explained in the next series; and perhaps also to unconscious correction for the illusion of the vertical.

The illusions in the diameter of the cylinder may be considered next. In Case 12 the vertical line was compared with the diameter of the horizontal cylinder and in Case 4 the horizontal line with the diameter of the vertical cylinder; the results are 10.4% and 12% respectively. This agrees very well with the tests of a similar nature upon the cube (See Cases 1 and 2), and as the same motives for illusion are present in both forms, the explanation given for one is adequate for the other. These percentages, then, are statements of the combined area and volume illusions for the diameter of the cylinder. In Case 10 the height of the plate was compared with the diameter of the horizontal cylinder. The result should give a statement of the volume illusion, but as recorded in Table IX it is zero. It will be remembered that this is one of the cases in which an allowance was made for the series of plates, and in this instance it would seem that this 5% was too great an allowance. Again, throughout Table IX there is brought out a general tendency for the illusion to be somewhat stronger when the dimensions compared are horizontal than when they are vertical. The reason for this will be given in the next series. It may be, then, for these two reasons, namely—too great an allowance for the error in the series of plates and the smaller illusion when the compared forms are vertical, that the volume illusion for the diameter of the cylinder is not brought out in Case 10. That it is present is indicated in two ways: first, by the fact that the 10.4% of Case 12 is too great for the area illusion alone, consequently something of the volume illusion must be involved; and second, when the width of the plate is compared with the diameter of the



vertical cylinder (Case 6) the volume illusion for the diameter of the cylinder is 6%. If the volume illusion is 6% for the diameter of the cylinder, then the area illusion for the diameter of the cylinder in Cases 12 and 4, which represent the combined effect of the two illusions, is 4.4% and 6% respectively for the two cases.

In connection with Series II it was found that the diameter of the cylinder was overestimated by 8%, and this illusion was termed the illusion of cylinder diameter. It is evident that the volume illusion of the present series and the illusion of cylinder diameter of Series II are precisely the same illusion; in other words, the overestimation of cylinder diameter is due to the volume illusion.

If the measurements upon the length of the drawn cylinder are compared with the corresponding measurements upon the length of the metal cylinder, it will be observed that the same general laws hold for the one as for the other. (Compare Cases 13 and 3, 17 and 11, 15 and 5, 19 and 9). The diameter of the drawn cylinder is also judged to be practically the same as the diameter of the metal cylinder. (Compare Cases 18 and 12, 14 and 4, 20 and 10, 16 and 6). These facts show that the same illusions are present in both the real and the drawn cylinders. In other words, the illusions persist as long as the form is perceived as a cylinder, although in reality it may not be a cylinder. The significance of this will be brought out in the next series.

The tests considered in the foregoing discussion were made with the forms on a level with the eyes of the observers. Corresponding measurements were made upon some of the forms when these were placed thirty degrees below the line of vision. For the height of the plate the angle seems to make no difference in the results. (Compare Cases 26 and 28, and 25 and 27). The same may be said of the cube (Cases 1 and 23, and 7 and 24), and also for the length of the cylinder (Cases 3 and 21, and 5 and 22). When measurements are made upon the diameter of the

cylinder in this position, a Müller-Lyer effect should appear causing an underestimation of the diameter. For some reason this is not brought out in Case 29, but it appears in Case 30. (Compare Cases 4 and 29, and 6 and 30). On the whole the measurements upon the forms in this second position support those made upon the forms in the first position.

The illusions involved in the standard forms in this series are shown in the analysis of them given in Table VII. The following general conclusions may be drawn:

The height of the plate is overestimated; this is the area illusion.

The area illusion also appears in the height and width of the cube.

It also appears in the length and diameter of the cylinder and in the length and diameter of the drawn cylinder.

In addition to the area illusion, the height and width of the cube are overestimated. This is the volume illusion.

The volume illusion appears in the length and diameter of the cylinder and it is also present in the length and diameter of the drawn cylinder.

The Müller-Lyer illusion sometimes affects the perception of the diameter of the cylinder.

#### *Series IV*

Series IV followed closely upon Series III, the records being made in January, 1901. The aim of the series was three-fold: first, to obtain measurements upon the illusion of the vertical for some typical geometrical forms; second, to test an hypothesis which would explain the volume illusion; and third, to study the illusions in some new forms. In addition to the plate, cube, cylinder, and drawn cylinder of Series III, the standard forms consisted of a sphere, a disk, a circle, a drawn square, and an ellipse. (See Fig. 1, Forms 4, 8, 12, 13, and 14, respectively). Each



form was compared with lines; the width or horizontal dimension of each was compared with a horizontal line, and the height or vertical dimension with a vertical line and a horizontal line.

The same method as for the lines in the previous series was employed. Six different series of lines were used with which to compare the standards, thus providing against the observer's selecting from memory. The backgrounds upon which the lines were drawn were also frequently turned so that the position of the top and bottom was reversed; and a different series of lines was used for each determination. The relative position of the standard and compared forms was the same as in the previous series. Two determinations were made for each of thirty-two cases with each of twenty observers.

The list of cases introduced into the series and the positions of the standard and compared forms is found in Table X.

In Table XI the signs for the several illusions which influence the records are given, and also the percent of illusion. In this table allowance was made for the 1% error due to the series of lines as explained in the previous series. The interrogation point is placed where there is some doubt as to the presence of the illusion. The difference in the signs for the illusion of the vertical will be explained later. The discussion of the series is based chiefly on Table XI.

In Table XII the individual estimates of the twenty observers are given. There were two trials for each case with each observer and the average of these forty trials is given, together with the amount of the illusion in millimeters and in percentages; also the %D. The individual mean variations are not given, since they can easily be made out from the table. In this table no deduction is made for the 1% error due to the series of lines.

The twenty observers in this series were selected from



TABLE X.

<i>Case</i>	<i>Dimension of Standard Form Measured</i>	<i>D. St. F.</i>	<i>D. Comp. Line</i>	<i>No. in Fig. 1</i>
1	Height of plate	Vertical	Vertical	1
2	Width of plate	Horizontal	Horizontal	1
3	Height of plate	Vertical	Horizontal	1
4	Height of cube	Vertical	Vertical	2
5	Width of cube	Horizontal	Horizontal	2
6	Height of cube	Vertical	Horizontal	2
7	Height of set-in cube	Vertical	Horizontal	2
8	Width of set-in cube	Horizontal	Horizontal	2
9	Height of set-in cube	Vertical	Vertical	2
10	Diameter of sphere	Horizontal	Horizontal	4
11	Diameter of sphere	Vertical	Horizontal	4
12	Diameter of sphere	Vertical	Vertical	4
13	Diameter of disk	Vertical	Vertical	8
14	Diameter of disk	Horizontal	Horizontal	8
15	Diameter of disk	Vertical	Horizontal	8
16	Long axis of ellipse	Vertical	Vertical	14
17	Long axis of ellipse	Vertical	Horizontal	14
18	Long axis of ellipse	Horizontal	Horizontal	14
19	Height of drawn square	Vertical	Vertical	13
20	Width of drawn square	Horizontal	Horizontal	13
21	Height of drawn square	Vertical	Horizontal	13
22	Diameter of circle	Vertical	Vertical	12
23	Diameter of circle	Horizontal	Horizontal	12
24	Diameter of circle	Vertical	Horizontal	12
25	Length of cylinder	Horizontal	Horizontal	3
26	Diameter of cylinder	Vertical	Horizontal	3
27	Length of cylinder	Vertical	Horizontal	3
28	Diameter of cylinder	Horizontal	Horizontal	3
29	Length of drawn cylinder	Horizontal	Horizontal	15
30	Diameter of drawn cylinder	Vertical	Horizontal	15
31	Length of drawn cylinder	Vertical	Horizontal	15
32	Diameter of drawn cylinder	Horizontal	Horizontal	15

*D. St. F.*, the direction of the dimension measured in the standard form.

*D. Comp. Line*, the direction of the line with which the standard was compared.

The number of the form in Fig. 1 is given in the last column.

TABLE XI.

<i>Case</i>	<i>A</i>	<i>Vl</i>	<i>C-L</i>	<i>M-L</i>	<i>V</i>	<i>% Il</i>
1	+	0	0	0	—	+ 2.5 (+3.5)
2	+	0	0	0	0	+ 7.7
3	+	0	0	0	+	+13.0
4	+	+	0	0	—	+ 5.0 (+7.4)
5	+	+	0	0	0	+10.4
6	+	+	0	0	+	+14.0
7	+	+	0	0	+	+14.0
8	+	+	0	0	0	+ 9.5
9	+	+	0	0	—	+ 6.0 (+7.5)
10	+	+	0	—?	0	+ 9.5
11	+	+	0	—?	+	+10.4
12	+	+	0	—?	—	+ 2.5 (+7.5)
13	+	0	0	—?	—	— 2.0 (+3.0)
14	+	0	0	—?	0	+ 6.0
15	+	0	0	—?	+	+ 7.0
16	+	0	0	—	—	— 3.6 (—0.2)
17	+	0	0	—	+	+ 6.0
18	+	0	0	—	0	+ 3.4
19	+	0	0	0	—	+ 3.4 (+6.9)
20	+	0	0	0	0	+ 9.5
21	+	0	0	0	+	+12.0
22	+	0	0	—?	—	— 1.0 (+4.0)
23	+	0	0	—?	0	+ 6.0
24	+	0	0	—?	+	+ 7.0
25	+	+	+	0	0	+16.5
26	+	+	0	0	+	+15.0
27	+	+	+	0	+	+20.0
28	+	+	0	0	0	+12.0
29	+	+	+	0	0	+17.4
30	+	+	0	0	+	+13.0
31	+	+	+	0	+	+18.3
32	+	+	0	0	0	+11.3

*A*, area illusion.

*Vl*, volume illusion.

*C-L*, cylinder-length illusion.

*M-L*, Müller-Lyer illusion.

*V*, illusion of the vertical.

*% Il*, percent of illusion.

1% for each case was deducted from the results as given in Table XII to eliminate the error due to the series of lines.

TABLE XII.

	<i>Case</i>	<i>1</i>	<i>2</i>	<i>3</i>	<i>4</i>	<i>5</i>	<i>6</i>	<i>7</i>	<i>8</i>	<i>9</i>	<i>10</i>	<i>11</i>	<i>12</i>	<i>13</i>	<i>14</i>	<i>15</i>	<i>16</i>
Obs	1	119	129	129	119	134	129	134	134	139	129	124	114	109	114	119	114
		119	129	129	124	139	129	134	134	124	119	119	114	109	114	114	109
	3	124	129	124	109	119	119	124	114	124	119	129	119	109	119	119	104
		134	134	139	124	134	144	134	124	124	139	134	124	139	139	144	129
	5	114	109	129	114	124	109	114	119	109	104	109	104	104	104	109	94
		104	104	104	104	109	104	104	109	104	99	104	94	94	99	99	94
	2	104	119	119	114	134	129	124	114	109	119	114	109	104	109	114	104
		104	114	114	104	114	119	114	114	104	114	119	104	104	114	114	104
	7	149	129	134	124	134	144	144	134	129	124	134	129	114	124	134	124
		124	129	144	134	134	144	139	134	139	129	134	124	119	124	124	119
	4	134	129	139	129	134	144	139	134	129	124	134	129	114	129	129	104
		124	129	134	124	134	144	144	129	134	129	129	124	119	119	134	109
	6	124	129	134	134	119	134	139	134	129	129	134	129	129	129	134	104
		119	114	134	124	134	139	124	129	124	129	124	129	114	119	129	109
	9	109	114	149	129	124	149	144	124	114	139	144	124	109	134	124	109
		109	129	139	114	139	139	139	134	119	134	144	114	109	129	134	109
	8	119	119	129	114	124	124	129	124	119	114	114	114	114	124	124	124
		119	129	134	129	129	129	124	134	134	129	129	124	119	124	129	119
	10	114	129	134	119	129	124	129	124	114	129	124	109	109	124	124	109
		114	129	134	119	134	139	139	129	129	129	129	114	114	134	129	114
	12	124	139	139	139	129	134	134	134	124	134	139	129	124	144	124	129
		129	144	139	139	139	144	139	144	124	144	139	129	134	139	134	129
	14	124	139	129	139	129	139	144	144	139	149	149	149	129	134	144	124
		134	144	149	139	144	144	144	144	139	149	149	149	139	149	144	134
	11	109	119	119	119	124	129	129	119	114	129	129	119	109	124	119	109
		129	129	144	129	134	134	139	129	129	124	129	124	119	129	119	114
	16	129	114	134	119	119	134	124	119	114	129	129	114	114	129	124	109
		119	129	134	119	119	129	134	124	119	129	129	124	114	124	129	109
	18	114	119	119	119	119	114	124	124	114	119	129	114	109	119	124	114
		114	124	124	114	129	119	119	124	114	124	114	114	114	109	126	104
	13	119	119	119	114	114	119	129	119	114	129	124	119	104	114	109	104
		114	114	119	119	119	124	129	124	124	124	119	114	114	114	114	104
	20	109	134	129	114	124	129	134	134	124	134	134	109	114	134	129	114
		119	129	129	124	129	129	134	124	119	129	134	114	114	134	124	114
	15	104	114	124	104	109	124	129	114	114	109	114	94	94	114	114	109
		104	114	124	114	114	124	119	114	114	109	114	94	104	109	114	94
	22	114	114	114	119	129	139	129	119	119	114	119	109	109	99	114	104
		114	114	124	114	119	124	119	119	114	114	114	109	104	109	104	114
	24	109	114	119	119	129	129	134	129	119	124	119	119	109	109	109	104
		114	129	139	119	129	129	129	129	119	129	129	119	104	119	124	109
Ave		118	124	130	121	127	131	131	126	122	126	127	118	113	122	123	111
Il		4	10	16	7	13	17	17	12	8	12	13	4	—1	8	9	—3
%Il		3.5	8.7	14	6	11.4	15	15	10.5	7	10.5	11.4	3.5	—0.9	7	8	—2.6
%D		7	7	7	6	6	7	6	6	7	7	8	8	7	9	7	8

Where no sign is given in the Il and the %Il columns the plus sign is understood. Other notation same as in Table I.



TABLE XII. Continued.

<i>17</i>	<i>18</i>	<i>19</i>	<i>20</i>	<i>21</i>	<i>22</i>	<i>23</i>	<i>24</i>	<i>25</i>	<i>26</i>	<i>27</i>	<i>28</i>	<i>29</i>	<i>30</i>	<i>31</i>	<i>32</i>	<i>Obs</i>
109	109	114	119	129	109	114	119	134	139	139	129	139	129	134	114	1
114	114	119	119	119	109	129	114	129	134	134	134	134	119	129	124	
124	129	124	119	114	124	124	139	139	144	149	144	144	139	139	134	3
129	129	129	139	134	124	134	129	139	149	139	139	134	134	129	129	
99	99	109	114	114	94	99	104	114	114	119	99	129	104	119	114	5
104	104	109	104	114	94	119	104	124	104	124	109	124	99	114	99	
114	109	109	119	109	109	119	119	119	119	134	119	129	119	134	119	2
124	119	109	114	119	104	114	114	124	119	129	124	129	119	129	119	
129	124	119	124	134	104	124	124	134	149	149	149	134	129	149	129	7
129	124	134	134	139	119	119	119	149	144	149	139	134	144	144	129	
119	114	119	134	139	114	129	129	139	134	144	129	144	139	149	129	4
119	114	129	134	139	124	129	129	144	134	144	129	144	139	149	139	
119	119	139	129	139	119	129	129	139	139	144	134	134	139	149	129	6
129	114	104	129	144	119	129	129	134	139	144	134	144	129	144	129	
119	119	109	139	144	114	129	139	139	144	149	134	144	134	144	134	9
129	129	119	129	139	109	129	129	144	134	149	134	134	129	144	129	
124	119	119	124	129	119	129	124	139	129	129	124	139	134	129	124	8
129	129	119	134	129	114	124	124	129	134	134	129	129	129	134	134	
124	124	114	124	129	114	124	119	119	124	129	129	134	129	134	129	10
124	129	124	129	134	114	129	129	134	134	144	129	134	134	129	129	
134	134	124	139	134	119	129	129	139	119	144	129	144	129	139	124	12
139	139	129	139	129	129	134	134	134	139	144	134	139	119	134	134	
124	134	134	144	149	129	134	144	144	149	144	139	144	139	144	144	14
149	139	139	144	149	134	139	144	149	149	149	149	149	144	149	139	
124	114	129	129	129	114	129	124	139	144	139	144	139	144	144	144	11
129	129	119	129	129	124	134	134	149	139	144	139	134	144	144	144	
114	114	119	114	129	114	114	114	134	129	144	124	129	129	129	129	16
129	114	129	129	134	114	114	119	134	129	134	129	134	129	139	124	
124	109	109	114	124	104	109	114	129	134	129	134	129	119	119	129	18
109	119	109	129	124	109	124	119	129	124	129	129	129	124	124	124	
109	109	114	109	114	114	119	114	119	114	124	119	144	129	124	124	13
114	114	114	109	114	114	114	129	129	124	129	119	124	119	129	129	
129	124	119	134	134	114	119	124	134	139	144	134	139	139	134	144	20
129	119	119	139	134	114	129	129	144	139	139	139	144	149	144	134	
109	114	104	119	124	104	114	119	129	124	134	119	119	114	134	114	15
114	109	104	114	119	109	114	114	129	124	129	114	124	119	134	114	
114	119	114	119	124	114	104	114	129	114	139	124	134	124	134	124	22
129	124	114	114	124	114	109	109	129	124	139	109	129	129	144	134	
114	114	119	119	114	104	104	104	124	119	129	114	134	139	134	124	24
119	114	114	124	134	104	114	114	134	129	139	124	139	134	129	139	
122	119	119	126	129	114	122	123	134	132	138	129	135	130	136	128	
8	5	5	12	15	0	8	9	20	18	24	15	21	16	22	14	
7	4.4	4.4	10.5	13	0	7	8	17.5	16	21	13	18.4	14	19.3	12.3	
7	6	6	5	7	6	7	7	6	8	6	7	7	7	7	6	

TABLE XIII.

<i>Dimensions of Standard Forms Measured in Terms of Lines</i>		<i>Series III 12 Observers</i>		<i>Series IV 20 Observers</i>	
	<i>Direction</i>	<i>Case</i>	<i>% Il</i>	<i>Case</i>	<i>% Il</i>
Height of plate	Vertical	26	+ 7.0	1	+ 2.5
Height of cube	Vertical	1	+12.0	4	+ 5.0
Width of cube	Horizontal	2	+14.0	5	+10.4
Length of horizontal cyl'r	Horizontal	11	+18.3	25	+16.5
Diameter of vertical cyl'r	Horizontal	4	+12.0	28	+12.0
L'gth of horiz'l drawn cyl'r	Horizontal	17	+20.0	29	+17.4
Diam. of vert'l drawn cyl'r	Horizontal	14	+12.0	32	+11.3

the class in elementary psychology. The degree of knowledge of the illusion possessed by each observer was so nearly the same that a discussion of the observers individually will be unnecessary. The subject of visual illusions had been studied in class but a short time before and the results bring out a tendency to allow for the illusion of the vertical in those cases in which the two compared forms were both vertical. The illusion of the vertical was the only illusion studied in class which would apply to the present series. The reaction of the observers against this illusion and its significance will be considered later.

Seven of the cases were repeated from Series III. The results of these cases are given in Table XIII to facilitate a comparative study. The two series were made at different times and with a quite different set of observers but the methods of procedure were the same. It is readily noticed that, exclusive of the measurements upon the height of the cube and plate, there is a very satisfactory agreement in the results for the two series. The explanation of the two exceptions just noted will be given subsequently. Had there been a considerable disparity in the results, this could have been explained by the fact that the force of the same illusion often varies greatly with different individuals. Since, however, the results are in close agreement, this may be taken as an indication of the reliability of the tests

and the relative constancy of the illusions involved. The existence of the illusions as they are indicated for the forms in Table XIII, may be regarded as established beyond reasonable doubt. These forms have been systematically discussed under Series III, and a repetition of that discussion is therefore unnecessary here.

One of the main purposes of Series IV was the study of the illusion of the vertical. To obtain measurements of this illusion in the various forms, the same dimension, alternately in the vertical and in the horizontal position, was compared with a horizontal line; the difference between the two results gave the desired measurements. The illusion as determined for the different forms is as follows:

The plate	5.2% Case 3 minus Case 2
The set-in cube	4.5% Case 7 minus Case 8
The cube	3.6% Case 6 minus Case 5
Length of the vertical cylinder	3.5% Case 27 minus Case 25
Diameter of the horizontal cylinder	3.0% Case 26 minus Case 28
Diameter of the horizontal drawn cylinder	2.7% Case 30 minus Case 32
Long axis of the ellipse	2.6% Case 17 minus Case 18
The drawn square	2.5% Case 21 minus Case 20
Diameter of the disk	1.0% Case 15 minus Case 14
Diameter of the circle	1.0% Case 24 minus Case 23
Diameter of the sphere	0.9% Case 11 minus Case 10
Length of the vertical drawn cylinder	0.9% Case 31 minus Case 29

The results indicate that the illusion of the vertical varies with the form; in general, excluding the circle and related forms in which it is checked, it is stronger for the less complex forms. The line was not studied in this series but in the other series the illusion of the vertical is on the whole about 1% stronger in the line than in the plate. The line with an illusion of 6% would therefore head this list of forms which are arranged in the order of the strength of the illusion. These figures are low because the observers had just heard the lectures upon geometrical illusions, as was stated, and the subject was fresh in their minds. The



average is also lowered by the exceptional reaction of a few against the illusion.

A most conclusive, systematic corroboration of this law of the variation of the illusion of the vertical with the form is found in the fact that throughout Series IV the results are smaller when both the standard and compared forms are vertical than when they are both horizontal. This is seen by comparing Cases 1, 4, 9, 12, 13, 16, 19, and 22, in which the forms were vertical, with Cases 2, 5, 8, 10, 14, 18, 20, and 23, in which the forms were horizontal. In the investigation up to this point, whenever the illusion of the vertical entered into the standard and the compared forms, its effect was regarded as eliminated from the results. This, however, appears in view of the present series, not to have been justifiable. Now, it is readily seen that when the vertical line is compared with the height of some standard form, for which the force of the illusion of the vertical is less than for the line, not all of the illusion of the vertical is eliminated from the result; only so much is eliminated as occurs in the standard form and there is still present the amount by which the illusion in the line is greater than in the standard form. For instance, the illusion of the vertical is 6% for the line, and, according to the present series, it is 1% for the circle. There is a difference of 5% in its force for the two forms. The consequence of this difference is that a line 5% shorter is selected as equal to the height of the circle, whereas, if the illusion were of the same strength for the two forms, so far as the illusion of the vertical alone is concerned, a line of the same height as the vertical diameter of the circle would be selected. The amount by which the illusion of the vertical for a given form is less than the same illusion for the line must be added to the result of the comparison of the vertical line with the height of that form. In order to eliminate the effect of the illusion of the vertical when two vertical dimensions are compared, it is neces-

sary to introduce corrections for the variation with form. More specifically, in Case 4, in which the height of the cube was compared with a vertical line, 2.4% must be added to the result, this 2.4% being the difference between the illusion of the vertical for the cube, (3.6%), and for the line, (6%). Case 4 then reads 7.4%. In like manner, 1% must be added to Case 1, 1.5% to Case 9, 5% to Case 12, 5% to Case 13, 3.4% to Case 16, 3.5% to Case 19, and 5% to Case 22. After these additions have been made the cases read as indicated in the parentheses in Table XI.

The records of Series III also indicated a difference in the force of the illusion of the vertical for the standard and compared forms and attention was called at the time to the fact that not all the illusion of the vertical was eliminated from the result. (Table IX. Cf. Cases 13 and 17, 3 and 11, etc.).

The variation with form introduces very grave complications in measurement. There is need of extensive experiments to determine the general laws for this variation more comprehensively and in detail in order that it may be taken into consideration in all measurements in which it is involved.

If this difference in the force of the illusion of the vertical for the standard and compared forms were the only disturbing factor, then, due allowance having been made for it, the measurements taken when these forms were both in the vertical position should agree with those taken when both the standard and compared forms were in the horizontal position. That is, Case 1 should agree with Case 2, Case 4 with Case 5, Case 9 with Case 8, Case 12 with Case 10, Case 13 with Case 14, Case 16 with Case 18, Case 19 with Case 20, and Case 22 with Case 23. The differences in the results for these eight pairs of cases are respectively, 4.2%, 3%, 2%, 2%, 3%, 3.6%, 2.6%, and 2%. An adequate explanation for these residuals may be found in the hypothesis that in all those cases in which two vertical distances were



compared, the observers made an allowance for the illusion of the vertical by selecting a shorter line, and this allowance may have been in either the standard or the compared form.

There has frequently been occasion to note, in the course of this research, that an observer often makes an allowance for, or reacts against, a known illusion without being in the least conscious of such a reaction or allowance. Careful questioning of each observer in this series met with the response that no allowance had been made consciously. But that such allowance did play a part is most clearly brought out in the cases noted. It is important to guard against this unconscious allowance for an illusion, although there is often no means of determining whether or not it is present. It is well to ascertain, if possible, the exact amount of preparation or foreknowledge of the observer; the more naive he is the better. The fact that introspection reveals no trace of this unconscious allowance points to the need of experimenting by objective methods upon observers in a naive state of mind.

A characteristic feature of this unconscious reaction against an illusion is the fact that there is often exhibited in it a lack of logical consistency. For instance, throughout Series IV the observers apparently made allowance only in those cases in which two vertical distances were compared. (Cases 1, 4, 9, 12, 13, 16, 19, and 22). The allowance, if made at all, should have been made in those cases in which the standard form was vertical and the compared form horizontal. This would tend to show that the allowance was not consciously made. The principle that vertical distances are overestimated was familiar to the observers and this principle was more strongly suggested in those cases where the allowance was made than in any of the other cases. This inconsistency was due partly to the observers' lack of training in the experimental method. This unconscious allowance was not made by every individual observer in Series IV, but the averages of the estimates of the ob-



servers as a class indicate plainly the presence of the general tendency.

In connection with Series III, a possible explanation of the volume illusion was suggested by the comparison of the judgments upon the plate and the cube. The forms were placed before similar backgrounds, and as only one face of the cube was visible and this of exactly the same size as the plate, the objective conditions in the perception of the two forms were almost identical. The side of the cube was judged to be larger than the plate and this was explained as being due to the volume of the cube. In every normal perception there is a subjective and an objective element. Since the objective conditions in the two perceptions mentioned were so similar, it is plain that the volume illusion must be explained from the subjective side of the perception. Accordingly a crucial test of it was introduced, based upon the following hypothesis, stated in terms of representative forms: The face of a cube looks larger than a square plate of the same size on account of the association of volume with the former. That is, experience has taught the observers that voluminal objects are larger than those without volume and his judgments are influenced by this knowledge. From this point of view, then, the volume illusion is not due to physiological causes, but is of entirely subjective origin and to be classed as an illusion of association.

The experiment for the testing of this theory was arranged according to the following plan: It is plain that if objectively the plate and cube were indistinguishable and the observer had no means of knowing that two separate forms were being studied, the results of a sufficient number of trials upon each of the two forms, other things being equal, would be the same. If, on the other hand, the same tests were made with the forms still objectively indistinguishable but with this subjective difference that the observer knew when he was looking at the plate and at the cube, any difference between the results for these two forms would be

due to this fact. The nature of this difference in the subjective element in the perception of the two forms is indicated by a study of the forms themselves; the plate has area alone, the cube has area and volume. The two forms differ in no other respects. The area of the face of each is directly perceived by the sense of sight; the volume of the cube is not, but the observer knows that he is looking at a cube. The idea of volume is present in the form of the judgment "there is more to the cube than to the plate" and the effect of this is brought out in the experiments.

The height of the plate was compared with a vertical line in Case 1 and with a horizontal line in Case 3, while in Case 2 the width of the plate was compared with a horizontal line. Cases 9, 7, and 8 respectively represent parallel measurements upon the cube. The cube was closely fitted into an opening in a background, like the one upon which the plate was fastened, so that only the front face was visible and this was flush with the plane of the background. Placed in this way the plate and the cube could not be distinguished by the observer. But when each form was presented to him he was told whether he was looking at the plate or at the cube. In fact he saw the cube adjusted in place each time. In this way the conditions necessary to test the theory that the volume illusion is due to the idea of volume were met.

When in Case 1 the height of the plate was compared with a vertical line, the result was 2.5%. The result of the same measurement upon the set-in cube was 6%. There is a difference of 3.5% between the two cases. When the height of the plate is compared with a horizontal line, Case 3, the result is 13%. The same test upon the height of the set-in cube, Case 7, gave 14% as a result, or a difference of 1%. The result of the comparison of the width of the plate with a horizontal line (Case 2) is 7.7%, and the same for the set-in cube (Case 8) is 9.5% or a difference of 1.8%. These differences of 3.5%, 1%, and 1.8% represent the effect of the idea of volume. The objective conditions in the three

pairs of cases were the same. The subjective conditions differed in that in connection with the cube there was the idea of volume, but with the plate there was no such idea present and herein lies the reason that the cube was judged to be larger than the plate. The hypothesis above stated, then, stands a crucial test. The illusion of volume is an association illusion, due to the presence of the idea of volume. It does not necessarily follow from this that the illusion is due to any conscious judgment upon the part of the observer. On the contrary it is quite probable that in his experience the tendency to interpret voluminal objects as larger than those without volume has become an habitual and well fixed unconscious process.

The 3.5%, 1%, and 1.8% given above are, however, not expressions of the full force of the volume illusion. One of the chief conditions of the presence of the illusion is that the motives be unrestricted. The illusion is stronger when the image of volume in the form is clear than when it is vague.<sup>1</sup>

In both Series III and IV, it has been shown that the results upon the drawn cylinder are practically the same as those upon the metal cylinder. This fact is another strong proof that the volume illusion is due to the idea of volume. The drawn cylinder is perceived as a solid; that is, it is thought of as having volume. The idea of volume is present and accordingly the same illusions appear as in the regular cylinder; the volume illusion appears even though the volume of the form is only suggested.

So much for the volume illusion.<sup>2</sup> The area illusion has

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<sup>1</sup> The conditions of this test are parallel to those upon the weight illusion in which it was demonstrated that the illusion is stronger when the size of the object that is lifted is seen than when it is merely remembered. SEASHORE, *Measurements of Illusions and Hallucinations in Normal Life*. Studies from the Yale Psychol. Lab. 1895, III, p. 11.

<sup>2</sup> The presence of the volume illusion in the wires used in Series II may explain the discrepancy noted for those cases (Vid. p. 59).



been shown to be connected in some way with the presence of area in the form. The analogy between the conditions for the area illusion and the conditions for the volume illusion is complete; therefore the area illusion is an association illusion and of the same general nature as the volume illusion.

Turning now to the new forms, the sphere will be considered first. In Case 10 the horizontal diameter of the sphere was compared with a horizontal line, and in Case 12 the vertical diameter with a vertical line. The results are +9.5% and +2.5% respectively. These results should be compared with the corresponding measurements upon the width (Case 5), and height (Case 4), of the cube, the results of which are +10.4% and +5% respectively. It is seen that the illusions in the sphere are almost as strong as those in the cube. In this latter form the overestimation of the height and width was explained by the area and volume illusions. The sphere and cube are both typical geometrical solids and the nature of the area and volume illusions is such that they are applicable to all such objects. The overestimation of the size of the sphere is caused, then, by the area and volume illusions. However, in the sphere there may be involved something of the Müller-Lyer illusion with a tendency to bring about a slight underestimation of the diameter. It is possible that for this reason the measurements upon the sphere are somewhat smaller than those upon the cube. The sphere also has smaller area and volume than the cube and this may have reduced the force of the illusions slightly. The 9.5% of Case 10 is a statement of the combined area and volume illusions for the sphere, or better, it is the amount by which these illusions outweigh the Müller-Lyer effect. In Case 12 the result obtained is not representative of the full force of the illusions, for it is one of those cases in which the observers made allowance, and also one in which the illusion of the vertical was less for the standard than for the compared form.

The disk bears the same relation to the plate as the sphere to the cube. The area illusion would tend to cause an overestimation of the diameter of the disk. If the Müller-Lyer illusion enters, it has an opposite effect. As in the case of the sphere, in the measurements upon the vertical diameter, correction must be made for the allowance of the observers and also for the variation of the illusion of the vertical with the form. In Case 14, the result of which is +6%, there is obtained a statement of the amount by which the area illusion in the disk outweighs the Müller-Lyer illusion. The effect of the latter illusion cannot be determined from these records.

A measurement of the illusions in the ellipse is obtained from Case 18, where the horizontal line was compared with the long axis in a horizontal position. The result is +3.4%, which represents the amount by which the area illusion is stronger than the Müller-Lyer illusion.

The drawn square is but a variation of the plate and the same illusions are involved in it as in the plate. It was inserted into this series partly as a control upon the plate and partly to determine whether the mode of definition of the area had any effect upon the resulting illusions. The black area of the plate, being throughout in contrast to the background, was more clearly defined than the area of the drawn square which was merely limited by lines. By comparing Case 20 with Case 2, and Case 19 with Case 1, it is seen that the drawn square is judged to be about 1% larger than the plate, both in height and width. A possible explanation of this is that the apparent size of the plate is reduced by irradiation. The principle of irradiation has not been taken into account before as its effect was practically eliminated. There may also be physiological reasons for the difference, since the eye movements are quite different in the perception of the two forms.

The circle and the disk stand in the same relation to each other as the plate and drawn square. In Case 23 the

horizontal diameter of the circle was compared with a horizontal line; the result is the same as for the disk, +6%. For some reason the effect of irradiation is not brought out in the results.

The results of Series IV seem to warrant the following statements:

The main conclusions of Series III are corroborated by fresh evidence. (Table XIII).

The illusion of the vertical varies with the form, decreasing with the increase in the complexity of the form. (See order of forms and exceptions to this rule, p. 81).

Knowledge of an illusion causes the observer to introduce a correction for it unconsciously; such unconscious systematic allowance for the illusion of the vertical is demonstrated.

The volume illusion is shown to be an association illusion due to the idea of volume; the volume may be either real, or suggested, as in a drawing. The volume illusion varies with the prominence of the idea of volume.

By analogy, the area illusion is an association illusion due to the idea of area.

In the new forms, the area illusion and the volume illusion are demonstrated for the sphere, and the area illusion for the disk, ellipse, drawn square, and circle.

### *Series V*

The tests in this series were made in the summer of 1901. The purpose was to obtain, by different methods, further measurements upon the illusion of the vertical for the line, and also to determine the area and volume illusions in some new forms. The method employed in all but the first four cases was the same as that used in Series IV. The standard forms consisted of a line, a cone, a pyramid, a triangle, a cylinder, a cube, a plate, a cone and cylinder, a pyramid and cube, and a triangle and plate.



(Fig. 1, Forms 16, 7, 6, 5, 3, 2, 1, 11, 10, and 9 respectively).

The observers were five members of the class in experimental psychology during the summer session of the University, one graduate student (Obs. 6), and an assistant-professor (Obs. 2). With reference to the illusion of the vertical these observers were of the first type, but with reference to the area and volume illusions they were of the second type.

Seventeen cases were introduced into the series and eight determinations upon each case were made for each observer in the double fatigue order. The different cases are stated in Table XIV; in Table XV the signs of the illusions involved and the results are given. As before, 1% has been subtracted to eliminate the error due to the series of lines in Cases 5 to 11 and 15 to 17 inclusive. The averages of the eight trials for each observer are given in Table XVI.

One purpose of this series was to measure the effect of the methods employed upon the illusion of the vertical, and the first five cases were arranged to test this. The methods and results for these cases are as follows:

Case 1. Method of production: to indicate on a horizontal line a distance equal to the standard vertical line. Two lines one millimeter wide were drawn at right angles upon a large sheet of light colored cardboard. The vertical line was 114 mm. in length and the horizontal line much longer. The problem was to cover with a strip of cardboard as much of the extra length of this line as was necessary to make the portion in sight appear equal to the standard vertical line. The only illusion known to enter is the illusion of the vertical, with a tendency to produce an overestimation. The result is +3.5%.

Case 2. Method of production: to indicate on a vertical line a distance equal to the standard horizontal line. This is the reciprocal of Case 1, the standard line now being

TABLE XIV.

*Case*

- 1 To lay off distance on horizontal line equal to vertical line.
- 2 To lay off distance on vertical line equal to horizontal line.
- 3 To select horizontal line equal to vertical line.
- 4 To select vertical line equal to horizontal line.
- To select a horizontal line equal to:—
- 5 Length of line, vertical.
- 6 Altitude of cone, vertical.
- 7 Diameter of base of cone, horizontal.
- 8 Altitude of pyramid, vertical.
- 9 Base of pyramid, horizontal.
- 10 Altitude of triangle, vertical.
- 11 Base of triangle, horizontal.
- 12 Length of cone and cylinder, vertical.
- 13 Length of pyramid and cube, vertical.
- 14 Length of triangle and plate, vertical.
- 15 Length of cylinder, vertical.
- 16 Height of cube, vertical.
- 17 Height of plate, vertical.

horizontal; the method was otherwise the same. The effect of the illusion of the vertical is to produce an underestimation. The vertical line is made 2.6% too short. In these first two cases the observers probably introduced some correction for the illusion of the vertical, especially in Case 2, where by varying the vertical dimension more attention is called to the illusion.

Case 3. Method of selection: to select a horizontal line equal to the standard vertical line. Two lines were drawn at right angles upon each of twelve cards of a light tint and about 28 cm. square. One of the lines on each card was the standard, 114 mm. in length; the other line varied on the different cards by five-millimeter steps from 94 mm. to 149 mm. The observer was told to select the card upon which the two lines appeared equal, according to the method used in Series I. In this case the horizontal line selected should be too long; it was selected too long by 4.4%.

Case 4. Method of selection: to select a vertical line

TABLE XV.

<i>Case</i>	<i>A</i>	<i>VI</i>	<i>C-L</i>	<i>M-L</i>	<i>V</i>	<i>L</i>	<i>% II</i>
1	0	0	0	0	+	0	+ 3.5
2	0	0	0	0	—	0	— 2.6
3	0	0	0	0	+	0	+ 4.4
4	0	0	0	0	—	0	— 4.4
5	0	0	0	0	+	0	+11.0*
6	+	+	0	—	+	0	+16.0*
7	+	+	0	—?	0	0	+ 7.7*
8	+	+	0	—	+	0	+13.0*
9	+	+	0	—?	0	0	+ 8.6*
10	+	0	0	—	+	0	+11.0*
11	+	0	0	—?	0	0	+ 7.0*
12	+	+	+	—	+	—	+11.4
13	+	+	0	—	+	—	+12.0
14	+	0	0	—	+	—	+10.5
15	+	+	+	0	+	0	+20.0*
16	+	+	0	0	+	0	+16.5*
17	+	0	0	0	+	0	+15.0*

*A*, area illusion.

*VI*, volume illusion.

*C-L*, illusion of cylinder length.

*M-L*, Müller-Lyer illusion.

*V*, illusion of the vertical.

*L*, illusion of length.

*% II*, percentage of illusion.

\*In these cases an allowance of 1% has been made to eliminate the error due to the series of lines.

equal to the standard horizontal line. This repeats Case 3 with the positions of the standard and variable lines reversed. The illusion is 4.4%, as in Case 3.

Case 5. Method of selection: to select from the series of horizontal lines the one equal to the standard vertical line. The method of procedure was the same as in Series IV. From the result, which is +11%, it is seen that the apparent illusion of the vertical is much stronger with this method than with the two just described. The reason for this is brought out in connection with Series VII, where the effect of turning around from one form to the other is considered.



TABLE XVI.

	<i>Obs 1</i>		<i>3</i>		<i>5</i>		<i>2</i>		<i>4</i>		<i>6</i>		<i>8</i>		<i>Ave</i>					
<i>Case</i>	<i>E</i>	<i>d</i>	<i>E</i>	<i>d</i>	<i>E</i>	<i>d</i>	<i>E</i>	<i>d</i>	<i>E</i>	<i>d</i>	<i>E</i>	<i>d</i>	<i>E</i>	<i>d</i>	<i>E</i>	<i>d</i>	<i>Il</i>	<i>% Il</i>	<i>% D</i>	<i>C'e</i>
1	113	2	118	2	117	2	127	2	115	3	117	3	120	5	118	3	+ 4	3.5	1.3	1
2	111	2	111	4	110	1	112	3	110	3	108	2	114	3	111	2	— 3	2.6	0.9	2
3	117	2	119	0	118	2	121	2	119	0	120	2	118	2	119	1	+ 5	4.4	0.9	3
4	112	3	109	0	109	0	110	2	108	1	108	1	110	4	109	1	— 5	4.4	0.9	4
5	120	1	132	4	133	6	125	3	127	4	129	8	130	3	128	4	+14	12.0	1.3	5
6	125	6	135	5	145	2	131	3	136	2	133	7	128	8	133	5	+19	17.0	4.4	6
7	117	3	128	4	132	3	122	3	127	4	121	8	124	8	124	5	+10	8.7	3.5	7
8	117	3	135	4	142	3	128	3	133	6	130	5	123	5	130	4	+16	14.0	5.2	8
9	116	3	130	3	130	6	122	3	125	2	123	6	127	5	125	4	+11	9.6	3.5	9
10	118	5	134	4	131	3	127	3	140	3	129	6	120	6	128	4	+14	12.0	5.2	10
11	114	4	122	4	128	4	121	2	131	5	122	4	120	7	123	4	+ 9	8.0	3.5	11
12	264	3	234	3	268	3	256	3	256	3	260	12	237	9	254	5	+26	11.4	4.4	12
13	264	8	239	5	269	1	257	2	249	6	267	8	243	12	255	6	+27	12.0	4.4	13
14	255	8	234	7	262	11	255	2	250	8	269	5	242	10	252	7	+24	10.5	4.0	14
15	128	4	145	2	147	3	132	5	148	2	140	5	138	4	138	4	+24	21.0	5.2	15
16	122	3	138	4	142	6	132	3	143	2	128	4	138	5	135	4	+21	17.5	5.2	16
17	118	2	135	3	140	9	129	0	143	2	128	4	130	3	132	3	+18	16.0	5.2	17

Notation same as in Table I.

Another of the main purposes of this series was to study the area and volume illusions in some new forms. These forms will be considered in order.

In the cone the area and volume illusions enter, tending to produce an overestimation of its size. The Müller-Lyer illusion is in all probability very strong for the altitude, but it is uncertain whether or not it has any effect upon the diameter of the base. The +16% of Case 6, where the altitude was compared with a horizontal line, represents the final balancing of the illusion of the vertical (+), of the area illusion (+), of the volume illusion (+), and of the Müller-Lyer illusion (—). That is, the three illusions outweigh the fourth by 16%. In Case 7 the diameter of the base of the cone was compared with a horizontal line. The area and volume illusions enter and there is probably a little of the Müller-Lyer effect. The line was selected 7.7% too long as a result of these illusions.

The illusions in the pyramid correspond to those in the cone. When the altitude is compared with a horizontal line (Case 8), the illusions amount to 13%, and when the base is compared with the line the illusions amount to 8.6%. The altitude of the cone is judged to be 3% greater than the altitude of the pyramid, and the base of the cone 1% less than the base of the pyramid.

In Case 10 the altitude of the triangle was compared with a horizontal line. The illusion of the vertical enters to produce an overestimation and also the area illusion with a like tendency. The Müller-Lyer illusion is doubtless involved, causing an underestimation. The final result is +11%, which represents the amount by which the illusion of the vertical and the area illusion exceed the Müller-Lyer illusion. The base of the triangle was compared with the horizontal line in Case 11. The Müller-Lyer illusion, although much reduced, is probably present. The line is selected 7% too long, which represents the area illusion minus the Müller-Lyer illusion. The base of the triangle is judged to be 0.7% less than the base of the cone and 1.6% less than the base of the pyramid. These differences are due to the volume illusion in the cone and pyramid, but they do not represent the full force of these illusions, for the Müller-Lyer illusion is involved. This illusion is stronger for the base of the cone and pyramid than for the base of the triangle, as is shown by the fact that the force of the area illusion (Case 11) is about normal, but the volume illusion in the cone and pyramid is too small.

Determinations upon the cylinder, cube, and plate were repeated so as to have measurements upon them singly in the same series in which they appeared in combinations. By comparing Cases 15, 16, and 17 of this series with Cases 27, 6, and 3 respectively of Series IV, a very close agreement will be observed.

The cone was placed upon the cylinder and the total length of the two forms was compared with a horizontal

line in Case 12. The purpose of these tests upon some of the forms in combination was to bring out variations in the illusions as a result of combining the forms. It was also desirable for purposes of comparison to make analytical studies of forms which were in a way typical of some natural objects which were being studied. The result obtained for the cone and cylinder in combination represents the effect of at least five illusions, but one of which, the Müller-Lyer illusion, causes an underestimation. The form is overestimated on account of the illusion of the vertical, the area illusion, the volume illusion, and the illusion of cylinder length. It is also possible that a sixth illusion enters; this is the illusion of length, according to which double distances are overestimated, and its ultimate effect, if it is involved, is to cause the selection of a shorter line, (—). This illusion enters into any form in which there is a motive to bisection, and it is weakest for those forms which are most easily bisected.<sup>1</sup> In judging the length of such a form as the cone and cylinder, the observer naturally estimates the distance by halves, and by so doing he introduces the conditions under which the illusion of length operates, and the length of the form is then overestimated. But when he bisects the linear dimension of the standard form measured, he must also bisect the compared line, and immediately the illusion of length enters into this form also, and it is likewise overestimated. However, it is easier to estimate the standard form in halves than the compared form, for the former actually consists of two pieces placed one upon the other, while the latter is simply a straight line. The illusion of length, then, would be greater in the line, because it is bisected with greater difficulty, hence a shorter line must be selected to cancel the effect of the overestimation. The influence of the illusion of length is therefore represented by the minus sign. If the observers

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<sup>1</sup> SEASHORE and WILLIAMS, *Op. cit.*, p. 597.



did not bisect the standard form, *i. e.*, judge it in halves, the illusion of length did not enter. In making his judgments Obs. 2 followed this method and for him all six of the illusions enter in the perception of the cone and cylinder and the other double forms. The total height of the cone and cylinder is 228 mm., but the observers selected as equal to it a line of 254 mm., or one 11% too long. (No allowance for the series of lines has been made for these double forms, for it is probably less than 1% for these longer lines). The sum of the results taken upon the forms singly is 18% of 228; that is, when the cone and cylinder are in combination, there is a reduction of the total illusion. This is probably due in part to the presence of the illusion of length in the double form.

The illusions involved in the pyramid and cube when in combination are, with the exception of the illusion of cylinder length, the same as those in the cone and cylinder. The result for Case 13, where the length was compared with a horizontal line, is 27 mm., or +12% of the standard distance. The sum of the corresponding determinations for the forms taken separately is +15% of 228, or 3% more than the result for the forms when combined.

In the combined triangle and plate the number of illusions is reduced to four: the illusion of the vertical (+); the Müller-Lyer illusion (—); the area illusion (+); and the illusion of length (—). The total result is an over-estimation of 24 mm., or +10.5%, as against +13% when the measurements are made upon the single forms.

The mean variation for these double forms is greater than for the single forms because the conditions are more vague and indefinite.

A complete analysis of the illusions involved in the forms in Series V is found in Table XV. The results of this series justify the following general conclusions:

The apparent strength of the illusion of the vertical varies with the method employed.

The area illusion is demonstrated in the triangle and the combined triangle and plate; and the area and volume illusions are demonstrated in the cone, pyramid, cone and cylinder, and pyramid and cube.

The illusion of length is present in both the standard form and the compared line when there is a motive to bisection as in the double standard forms.

The total effect of the illusions in the length of the double forms (Fig. 1, Forms 9, 10, and 11) is not as great as the sum of the corresponding illusions in their components.

### *Series VI*

Series VI, the records of which bear the date of July, 1901, was planned to show the effect of distance upon some of the illusions studied in the previous series.

The observers were six young women, members of the class in elementary psychology in the summer session, and on the whole they may be classed as observers of the second type.<sup>1</sup> Four trials on each case were made by each observer.

The effect of distance upon the illusion of the vertical in the square, upon the illusions in the cylinder, and upon the illusion of the vertical in the line, was studied. A, B, and C, following, refer respectively to the square, the cylinder, and the line.

A. Case 2 of Series I and II was repeated with the observers standing at different distances. The problem was to find the square plate at a distance of 1 meter (Case 1); at 3 meters (Case 2); at 6 meters (Case 3); at 9 meters (Case 4); and at 12 meters (Case 5). The results are given in Table XVII, from which it is seen that the illu-

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<sup>1</sup> Observer 9 has astigmatic eyes. Right eye has 1.50 diopters of simple myopic astigmatism, axis 95 degrees; left eye, 1.50 diopters of simple myopic astigmatism, axis 98 degrees.

TABLE XVII.

	<i>Case 1</i> (1 meter)		<i>Case 2</i> (3 meters)		<i>Case 3</i> (6 meters)		<i>Case 4</i> (9 meters)		<i>Case 5</i> (12 meters)	
<i>Obs</i>	<i>E</i>	<i>d</i>	<i>E</i>	<i>d</i>	<i>E</i>	<i>d</i>	<i>E</i>	<i>d</i>	<i>E</i>	<i>d</i>
1	104	0	107	2	107	0	110	1		
3	107	2	110	3	111	1	115	1	113	1
5	110	1	109	1	110	3	109	1	108	1
7	108	2	108	1	110	2	108	1	111	3
9	103	1	103	2	105	1	104	0	108	2
11	102	4	100	1	104	2	104	2		
	—	—	—	—	—	—	—	—	—	—
Ave	106	2	106	2	108	2	108	1	110	2
<i>Il</i>	—8		—8		—6		—6		—4	
% <i>Il</i>	—7		—7		—5		—5		—3.5	
% <i>D</i>	2.6		2.6		2		2.6		1.8	

Notation same as in Table I.

TABLE XVIII.

	<i>Case 1</i> (1 meter)		<i>Case 2</i> (3 meters)		<i>Case 3</i> (6 meters)		<i>Case 4</i> (9 meters)		<i>Case 5</i> (12 meters)	
<i>Obs</i>	<i>E</i>	<i>d</i>	<i>E</i>	<i>d</i>	<i>E</i>	<i>d</i>	<i>E</i>	<i>d</i>	<i>E</i>	<i>d</i>
1	95	1	94	0	94	0	95	1		
3	101	3	102	1	106	1	103	2	107	2
5	98	3	98	2	99	3	102	3	105	2
7	105	2	105	2	108	2	104	2	108	2
9	98	3	98	3	98	2	99	3	102	0
11	89	0	88	4	92	4	94	2	94	2
	—	—	—	—	—	—	—	—	—	—
Ave	98	2	98	2	100	2	100	2.5	103	2
<i>Il</i>	—16		—16		—14		—14		—11	
% <i>Il</i>	—14		—14		—12		—12		— 9.6	
% <i>D</i>	3		3		4		3		3	

Notation same as in Table I.

sion of the vertical in the square of this size decreases with increase in distance, the illusion being twice as strong for the nearest as for the farthest distance.

B. Case 1 of Series I and II was repeated with the distance varied as for the plate above. For a detailed description of the test and the illusions involved in the cylinder,



the reader is referred to Series II. From the results, which are given in Table XVIII, it is seen that there is a tendency for the illusions in the cylinder to decrease with the increase in distance.

C. The purpose of this part of the series was to determine the effect of distance upon the illusion of the vertical for the line. The method of production was adopted and new apparatus devised in which the lines were represented by black watch springs. A piece of light manilla cardboard was tacked upon a frame 45 cm. by 90 cm. From the front side the ends of a section of a watch spring, 3 mm. in width, were pushed through two small slits 114 mm. apart. There was thus visible, from the front, a portion of the spring 114 mm. long, which was the standard line. The variable spring was pushed through from the back of a similar background and lay closely against the front side of it. The experimenter varied the length of the visible part of this spring as the observer directed until so much was exposed as was judged to be equal to the standard. The standard line was placed in front of the observer and a little below the level of the eyes. It was also in the horizontal position throughout the experiment. The background with the variable line rested upon an easel and was on a level with the eyes and at right angles to the line of regard. In Cases 1, 3, 5, and 7, the standard and variable lines were both horizontal, but the distances for the four cases were 3, 6, 9, and 12 meters respectively. In Cases 2, 4, 6, and 8, the distances were as above, but the variable line was vertical. The results are given in Table XIX.

It is a matter of common experience that an object at a distance appears to be smaller than one of the same size close at hand: hence in Cases 1, 3, 5, and 7, in which the illusion of the vertical is not involved, one would expect the sign of the result to be plus; that is, the farther line should be made longer in order to appear equal to the nearer one. It happens that the reverse of this occurs;

TABLE XIX.

	<i>Case 1</i>		<i>Case 2</i>		<i>Case 3</i>		<i>Case 4</i>		<i>Case 5</i>		<i>Case 6</i>		<i>Case 7</i>		<i>Case 8</i>	
	<i>(3 meters)</i>		<i>(3 m'rs)</i>		<i>(6 m'rs)</i>		<i>(6 m'rs)</i>		<i>(9 m'rs)</i>		<i>(9 m'rs)</i>		<i>(12 m'rs)</i>		<i>(12 m'rs)</i>	
<i>Obs</i>	<i>E</i>	<i>d</i>	<i>E</i>	<i>d</i>	<i>E</i>	<i>d</i>	<i>E</i>	<i>d</i>	<i>E</i>	<i>d</i>	<i>E</i>	<i>d</i>	<i>E</i>	<i>d</i>	<i>E</i>	<i>d</i>
1	118	3	101	2	105	1	91	5	105	9	84	1	101	2	87	5
3	119	3	115	9	130	11	126	5	131	5	122	10	134	5	132	10
5	106	12	107	5	105	7	103	5	98	8	101	5	98	7	105	10
7	117	9	110	1	110	11	99	3	105	6	99	1	95	8	93	4
9	104	8	102	8	92	6	106	5	96	10	101	6	99	5	107	5
11	110	5	103	6	114	9	99	10	113	5	107	2	116	8	113	6
<hr/>																
Ave	112	7	106	5	109	8	104	6	108	7	102	4	107	6	106	7
<i>Il</i>	—	2	—	8	—	5	—	10	—	6	—	12	—	7	—	8
% <i>Il</i>	—	2	—	7	—	4	—	8	—	5	—	10.5	—	6	—	7
% <i>D</i>	5		3		8		7		8		7		10.5		10	

Notation same as in Table I.

the signs of the results are minus signs. The explanation of this is that the observers consciously allowed for the variation in apparent size with distance.<sup>1</sup> They reasoned as follows: a line at a distance is really longer than it appears to be; in other words, a distant line which appears to be equal to a near line is really longer than the near line, therefore, if the distant line is to be actually equal to the near line, it must appear to be somewhat shorter, and the farther away the distant line is, the shorter it must be made. This reasoning was applied consistently, for there is a gradual increase in the amount of allowance which was made. The result for Case 1 is —2%; for Case 3, —4%; for Case 5, —5%; and for Case 7, —6%. These percentages are statements of this conscious allowance. In Cases 2, 4, 6, and 8, in addition to this conscious allowance for variation in apparent size with distance, the illusion of the vertical is involved with a tendency to cause the production of a line shorter than the standard (—). Thus there

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<sup>1</sup> This does not apply to Observer 3, whose records indicate that she made no allowance.

are in these cases two motives for making the compared or distant line too short. The results for the four cases are respectively  $-7\%$ ,  $-8\%$ ,  $-10.5\%$ , and  $-7\%$ . To secure a statement of the illusion of the vertical for the different distances, the amount of allowance which was made must be eliminated from these results:  $2\%$  must be deducted from Case 2,  $4\%$  from Case 4,  $5\%$  from Case 6, and  $6\%$  from Case 8. The illusion of the vertical in the line then is  $5\%$  for Case 2,  $4\%$  for Case 4,  $5.5\%$  for Case 6, and  $1\%$  for Case 8. The results for Case 8 are probably less reliable than the others, as the compared line could not be seen with distinctness at a distance of 12 meters.

The following statements may be made from the results of Series VI:

The illusion of the vertical in the square varies with distance; it decreases with the increase in distance.

The illusions in the cylinder also decrease with the increase in distance.

There is a tendency to make a conscious allowance for variation with distance in the apparent length of the line.

No appreciable variation of the illusion of the vertical in the line, with distance, is indicated.

### *Series VII*

In this series an analysis of the methods most frequently used was attempted. It was noticed that the illusion of the vertical for the line appeared to be larger than usual when the observer turned around ninety degrees to select from the series of lines one equal to the standard line. This is well illustrated in the first five cases in Series V. As the validity of all the conclusions drawn rested to some extent upon the reliability of the method employed, it was seen to be necessary to make a very careful study of the methods of production and selection used in the tests. The



illusion of the vertical for the line was taken as a medium for this study.

The six persons who served as observers were members of the laboratory class in experimental psychology in December, 1901. They had paid but little attention to illusions for about a year; they knew of the illusion but were not conscious of making allowance and they may therefore be classed as observers of the first type. The comment of one observer, that he had forgotten which way the illusion of the vertical ought to work in the various cases and so had considered it unsafe to try to allow for it, was a characteristic remark. When several different cases follow each other in rather rapid succession, the observer who has no practice in this experiment becomes, in a way, confused and usually abandons any attempt to react consistently against an illusion.

There were eight cases in the series and each observer made sixteen trials in each case. Half the number of trials, in the double fatigue order, were made at one sitting. In the first four cases the method of selection was employed.

Case 1. To select a horizontal line equal to the vertical standard line.

Case 2. To select a vertical line equal to the horizontal standard line.

Case 3. To select a horizontal line equal to the horizontal standard line.

Case 4. To select a vertical line equal to the vertical standard line.

The method employed in these four cases was the same as that used in Case 5 of Series V (p. 93). The observers turned upon a stool from one background to the other and selected from a series of lines one equal to the standard line. The lines were on a level with the eyes and one meter away.

In the remaining four cases the method of production was used.

Case 5. To produce a horizontal line equal to the vertical standard line.

Case 6. To produce a horizontal line equal to the standard horizontal line. The backgrounds fitted with watch springs as described for Series VI, were used. The standard and compared lines occupied the same position, relatively to each other, as in the first four cases; that is, the observer turned from one to the other.

Case 7. To produce a horizontal line equal to the vertical standard line.

Case 8. To produce a vertical line equal to the horizontal standard line.

Watch springs were again used for lines, but both the standard and variable lines were upon one background, which was 90 cm. square. Thus the effect of turning and the error due to the series of lines were eliminated. The two lines were at right angles to each other but the diagonal distance between the two adjacent ends was 114 mm.

The results for this series are given in Table XX, the study of which brings out several important facts.

The first of these to be considered is the effect of the series of lines upon the apparent length of the line. In

TABLE XX.

	<i>Case 1</i>	<i>Case 2</i>	<i>Case 3</i>	<i>Case 4</i>	<i>Case 5</i>	<i>Case 6</i>	<i>Case 7</i>	<i>Case 8</i>
<i>Obs</i>	<i>E d</i>	<i>E d</i>	<i>E d</i>	<i>E d</i>	<i>E d</i>	<i>E d</i>	<i>E d</i>	<i>E d</i>
1	127 8	111 5	120 4	114 4	122 6	119 3	119 3	107 3
3	124 5	106 3	115 3	112 3	127 4	116 3	121 3	110 2
2	125 7	112 5	118 4	115 4	124 4	118 4	120 5	110 5
5	124 4	117 2	121 4	120 4	119 3	116 3	117 2	117 2
7	122 3	110 2	116 2	113 1	119 3	115 2	118 3	103 2
9	123 6	111 4	118 5	115 3	124 4	115 3	120 2	109 2
	— —	— —	— —	— —	— —	— —	— —	— —
Ave	124 6	111 4	118 4	115 3	123 4	117 3	119 3	109 3
<i>H</i>	+10	— 3	+ 4	+ 1	+ 9	+ 3	+ 5	— 5
% <i>H</i>	+ 9	— 2.6	+ 3.5	+ 0.9	+ 8	+ 2.6	+ 4.4	— 4.4
% <i>D</i>	1	1.6	1.6	1.7	2	1.6	1	2.6

Notation same as in Table I.

connection with Series III, it was stated that when a line stands among others in a series it looks too short by 1% and proper allowance was made for this error in every series in which it occurred. This 1% was obtained from a comparison of Cases 1 and 5, and 3 and 6 of the present series. The standard line was vertical and the compared line horizontal in both Cases 1 and 5, but in the former instance the compared line was one of a series and in the latter it stood alone. All the other conditions were as nearly the same as it was possible to make them. The difference between the results for the two cases is an indication of the effect of the series of lines. The result for Case 1 is +9% and for Case 5 it is +8%; there is a difference of 1%. Cases 3 and 6 are parallel to these two except that the illusion of the vertical is not involved in either. In both instances the standard and compared lines are horizontal, but where the compared line is one of a series (Case 3), the result is 0.9% greater than when it is alone (Case 6). A line placed in such a series as this (see p. 60) then, looks too short by 1%, and a line longer by a corresponding amount is selected by the observers.

This error due to the series of lines may be termed an illusion of position; it enters as a result of the position of the line in a series. Only one line was fixated at a time but the other lines were in the indirect field and so influenced the line attended to. It is probable that this error in the series of lines varies with different individuals, with the position (whether vertical or horizontal) of the line, and with the distance apart and arrangement of the lines upon the background, but for want of time the effect of these various conditions could not be measured. The allowance of 1% has been made in all cases into which this error enters upon the assumption that the error due to the series of lines was of approximately the same force for the several conditions noted.



The effect of turning from one background to the other was also determined. Cases 5 and 7 were alike in all respects except that in the former the observer looked from one line to the other, while in the latter both lines were in the field of vision at the same time. The illusion in Case 5 is 3.6% greater than in Case 7, and this difference must in some way be due to the turning. In Case 3 the illusion of the vertical is not involved, but, after the elimination of the error due to the series of lines, there is a residual of 2.5%, which is another statement of the effect of turning. Still another measurement of this is obtained from Case 6, where it amounts to 2.6%. These three statements of the effect of turning from the standard to the compared line are gained from independent sources and they are all in the same direction. The error is not peculiar to one method, for the same tendency is brought out both by the method of selection (Case 3) and the method of production (Case 6). It cannot be explained upon the hypothesis that the turning increases the illusion of the vertical by giving greater freedom, because the illusion of the vertical is not involved in either Case 3 or Case 6, in which this error is about the same as for those cases in which the illusion enters. The general law that the second of two equal stimuli appears greater in intensity, as in the case of sounds and weights, does not apply, for the observer looked back and forth from one line to the other, and furthermore, the error is not in the right direction to be accounted for in this manner.<sup>1</sup> The effect of this illusion due to turning from one line to the other, is to increase the apparent length of the standard line; this is shown by the selection or production of a line longer than the standard as equal to it. It was thought best not to make any allowance throughout the several

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<sup>1</sup> The observer was allowed to turn back and forth and make several comparisons between the standard and the compared forms before giving his judgment. Therefore one cannot say that the standard was the first, and the compared form the second stimulus.

series for this error due to turning, as this was a constant element in all the tests and so has no effect upon the relative value of the results nor the conclusions drawn from them, other than to reduce slightly some of the percentages given.

It will perhaps be wondered why the result for Case 2, which is  $-3.6\%$  after the  $1\%$  has been subtracted for the error due to the series of lines, is not the same as for Case 1, since apparently it is only the reciprocal of it. Two reasons may be given for the lack of a closer correspondence in the two cases. One is that more attention is called to the illusion of the vertical when the vertical line is varied, and there is therefore a stronger tendency to allow for the illusion. A second reason is that the apparent length of the standard line is increased by turning, and this results in the selection of a longer compared line, thus reducing the apparent force of the illusion of the vertical in Case 2.

Case 8 was introduced as a check upon Case 7, the same illusion being involved in both cases. The method of production, in which neither the error due to the series of lines nor that due to turning was involved, was used. The results for the two cases agree; and moreover they agree also with Cases 4 and 3 of Series V, in which the same method of production was used but with different apparatus and other observers.

In the method of selection as most frequently used, it is found then that the error due to the series of lines is  $1\%$  (Case 3 minus Case 6); and that the effect of turning is  $3.6\%$  (Case 5 minus Case 7). By the method of production, in which these errors were not involved, the illusion of the vertical for the line is  $4.4\%$  (Cases 7 or 8). In Case 1, the illusion of the vertical, the error due to the series of lines and that due to turning are all involved. The result for this case is  $9\%$ , or an exact equivalent of the sum of these three tendencies ( $4.4\%$ ,  $1\%$ , and  $3.6\%$ ), the statements of which were obtained from sources entirely independent of Case 1.

The following conclusions may be drawn from this series:

The apparent length of the line is decreased when the line is one of a series like the one employed.

In turning ninety degrees around from the standard to the compared line, the apparent length of the standard is increased.

The illusion of the vertical is the same for either the method of production or the method of selection after the elimination of the error due to the series of lines and that due to turning. The variation shown in Series V is therefore only apparent.

### *Series VIII*

This series was arranged to determine the effect of practice upon the illusion of the vertical for the line.<sup>1</sup> In this form the illusion of the vertical seems to be the only illusion involved. The records are dated January, 1902. A series of one thousand tests was made upon each of three observers. There were ten periods of practice for each observer and one hundred trials were made at each period. These ten periods of practice occurred on successive days, except Saturdays and Sundays. Each period was about an hour long and with one or two exceptions the hour of the day was the same for each observer.

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<sup>1</sup> Professor Judd has made a study of the influence of practice upon the Müller-Lyer illusion. Several hundred measurements were made upon each of two observers with the result that the force of the illusion decreased rapidly with practice. The nature of this improvement with practice was regarded as "a change in the perceptual process, which change has taken place through repeated efforts to deal directly with the objects perceived." This perceptual process was regarded as "uninfluenced by expectation." *Psychol. Rev.*, 1902, IX, p. 27. The announcement of Professor Judd's conclusion in regard to the Müller-Lyer illusion was the immediate occasion for making these experiments upon the illusion of the vertical, although the study of the effect of practice was a part of the original plan.



Observer 1, a young lady, seemed to be entirely unaware of the existence of any illusion. She was a careful observer, wholly without practice, and her judgments were naive. At no time was there a suggestion made from which she could obtain any hint as to how she was progressing. At the beginning of the series she was told simply that the object of the experiment was to determine whether or not practice had any influence upon the accuracy of her judgments.

Observer 2, a man of extremely phlegmatic temperament, was an advanced student in psychology. He had studied illusions and had had some practice as an observer in some of the previous tests. He aimed to avoid recalling what he had studied in regard to illusions, and according to his own statement he made no allowance whatever. Each judgment was independent of all the preceding ones and appeared correct to him. He was not told anything about his records until all the trials were completed.

Observer 4 had the advantage of knowledge of the subject of illusions and also a considerable amount of practice as observer. He was not at all sure that the illusion would come out for him, but he endeavored faithfully to make judgments which appeared to him correct. He was told nothing about the results of any of the other observers nor of his own until all the trials were completed and his own introspective account had been written out. This account is given herewith.

“January 16, 1902. The comparison has been made by fixating the middle of each of the lines, thus bisecting them and comparing the four halves rather than the wholes.<sup>1</sup> I adopted this method because I had formed the habit in

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<sup>1</sup> On page 96 it was stated that when there is a motive for bisection the illusion of length enters. This does not have any effect upon these results as the standard and compared lines are bisected with equal effort and consequently the effect of the illusion of length is eliminated.

previous experiments and it seems to me to be the easiest and most accurate method of comparing. It did not obviate eye movements. I have felt no inclination to use units of measurement. I have imagined the horizontal line turned to the vertical position and superposed upon the standard line. In this act I have felt a strong inclination to make a correction of something over 6% which I estimate my normal illusion to be under these particular conditions. This 6% is only an estimate because I have never measured my illusion under these conditions before. The new factor here is the distance between the two lines. The method of adjusting the variable line is also new. The correction tends to enter irresistibly in the perceptive process. I am unable to tell whether I have made any correction or not. But, from my point of view, the conditions have been uniform throughout each period and throughout the whole series. If the results show any systematic variation in the progress of the experiment, I think it may be due to practice. I have asked the experimenter to mark certain records with which I felt especially well satisfied. There were marked differences in the certainty that I felt."

The method of production was employed by means of a new apparatus which proved exceedingly satisfactory. The lines to be studied were represented by watch springs as in the foregoing series. (Standard length, 114 mm.; width of spring, 2 mm). The observer adjusted the length of the compared line (the adjustable spring) by moving a lever. The lever was pivoted at a point 1 meter back of the background. From this point an arm extended to the back side of the background where it was connected with a sliding guide, to which one end of the spring was attached; the other arm of the lever was 2 meters long and came within convenient reach of the observer. This enabled the observer to make his own adjustments quickly and accurately, and eliminated the personal element of the experimenter.

In the statement of the results for each observer, the one

TABLE XXI. *Observer 1*

<i>1st day</i>		<i>2d day</i>		<i>3d day</i>		<i>4th day</i>		<i>5th day</i>	
<i>E</i>	<i>d</i>	<i>E</i>	<i>d</i>	<i>E</i>	<i>d</i>	<i>E</i>	<i>d</i>	<i>E</i>	<i>d</i>
139.7	4.3	135.5	2.6	145.2	1.8	138.6	2.4	142.4	2.8
132.7	3.1	137.8	3.4	143.6	2.4	142.3	4.1	142.3	2.3
134.2	4.4	136.7	1.1	144.3	2.0	141.3	2.5	141.8	3.0
138.6	2.9	134.1	3.5	142.3	2.4	137.8	4.6	137.2	2.6
136.3	4.0	134.9	4.5	140.3	2.3	135.4	3.2	138.0	1.2
134.9	3.1	134.7	2.5	136.5	2.5	137.2	3.8	141.3	2.1
138.2	4.2	134.2	3.0	138.1	2.9	139.6	2.4	138.9	2.1
138.1	4.3	134.6	2.2	138.8	3.2	138.9	3.1	137.1	1.9
140.3	3.5	135.5	2.5	141.3	2.1	137.7	2.7	137.1	1.3
140.5	1.3	135.2	1.9	141.8	2.8	135.4	2.8	136.3	2.1
Ave	137.4	135.4	2.7	141.2	2.4	138.4	2.2	139.2	2.2
% Il	20.2	18.4	23.7	21	21.9				

<i>6th day</i>		<i>7th day</i>		<i>8th day</i>		<i>9th day</i>		<i>10th day</i>	
<i>E</i>	<i>d</i>	<i>E</i>	<i>d</i>	<i>E</i>	<i>d</i>	<i>E</i>	<i>d</i>	<i>E</i>	<i>d</i>
143.3	1.7	140.6	2.2	142.9	2.4	137.6	1.6	141.8	2.0
142.1	2.1	138.6	2.6	143.9	2.5	138.1	2.3	141.5	2.5
140.3	3.5	132.8	1.0	138.1	2.1	137.9	1.3	139.8	2.0
141.9	1.9	136.3	2.5	136.0	1.2	138.7	1.9	137.4	2.0
137.9	3.0	135.8	1.2	135.2	1.4	137.0	1.8	135.8	1.6
141.8	1.6	138.0	2.4	134.8	2.6	137.0	2.2	134.4	2.0
139.3	2.5	137.2	2.6	137.5	1.3	134.5	2.7	139.1	3.3
136.7	1.7	134.3	0.9	133.8	2.4	138.1	3.1	137.2	3.0
140.0	1.8	136.3	1.7	133.6	2.2	136.2	1.8	137.3	1.9
138.0	1.2	132.7	1.7	136.0	1.6	137.6	2.0	135.9	2.3
Ave	140.2	136.3	1.9	137.2	2.0	137.4	2.1	138.0	2.3
% Il	22.8	19.3	20.2	20.2	21				

Final average, 1000 trials, 138 mm. or an illusion of 21%. Mean variation, 2%.

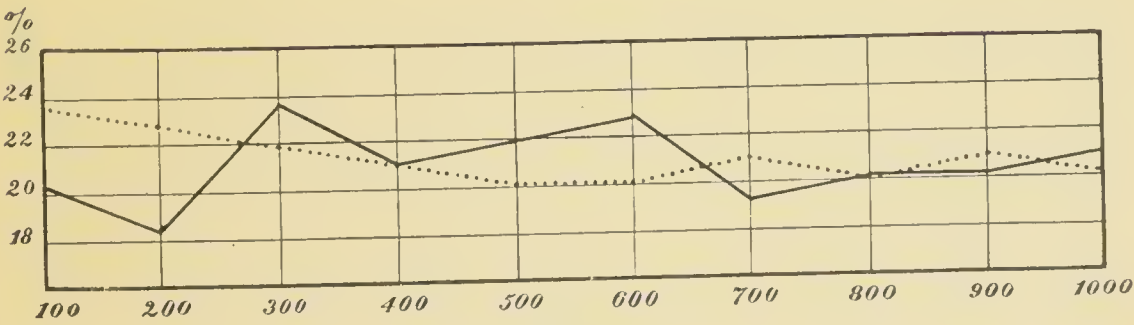


FIGURE 2



TABLE XXII. *Observer 2*

<i>1st day</i>		<i>2d day</i>		<i>3d day</i>		<i>4th day</i>		<i>5th day</i>		
<i>E</i>	<i>d</i>	<i>E</i>	<i>d</i>	<i>E</i>	<i>d</i>	<i>E</i>	<i>d</i>	<i>E</i>	<i>d</i>	
139.4	6.4	120.7	7.2	136.5	5.4	137.1	3.5	135.6	4.6	
142.0	3.6	126.1	4.7	140.7	3.1	138.4	2.0	134.6	3.0	
139.6	5.7	128.4	5.4	143.7	2.5	138.1	2.3	136.0	8.0	
146.0	3.2	126.1	2.5	144.3	3.9	142.6	1.2	145.7	3.3	
135.4	4.6	133.1	2.1	155.3	3.5	136.5	4.7	138.3	3.3	
138.0	6.2	134.0	4.0	145.0	8.4	144.6	6.6	142.8	4.4	
134.7	6.9	140.3	3.1	144.8	4.0	143.1	2.9	145.5	2.7	
138.0	5.2	138.3	4.5	141.4	3.0	139.9	5.1	142.2	3.0	
136.5	4.3	137.5	4.1	140.9	5.5	141.2	5.4	139.1	4.3	
135.6	6.0	137.4	4.4	143.9	3.9	143.7	3.9	135.9	1.3	
Ave	138.5	5.2	132.2	4.2	146.7	4.3	140.5	3.8	139.6	3.8
% <i>Il</i>	21.9	15.8		29		23.7		22.8		

<i>6th day</i>		<i>7th day</i>		<i>8th day</i>		<i>9th day</i>		<i>10th day</i>		
<i>E</i>	<i>d</i>	<i>E</i>	<i>d</i>	<i>E</i>	<i>d</i>	<i>E</i>	<i>d</i>	<i>E</i>	<i>d</i>	
139.8	2.2	137.2	3.4	141.7	2.7	135.7	1.7	136.0	1.2	
136.4	4.4	137.1	2.9	140.9	3.5	136.0	1.1	136.3	0.8	
140.7	2.9	136.9	1.7	138.3	3.3	134.4	1.6	134.5	1.1	
140.2	3.2	141.0	4.8	132.3	1.1	134.9	1.9	137.2	2.4	
142.0	4.4	139.2	3.6	132.8	1.4	135.5	0.9	134.9	1.9	
143.6	1.4	134.5	3.5	133.1	1.5	136.2	1.7	136.2	0.8	
140.3	2.3	136.0	3.0	131.0	3.6	139.8	1.8	137.8	2.0	
136.0	3.2	135.4	4.0	133.5	0.9	137.6	1.6	134.8	1.4	
139.1	3.1	138.3	2.3	134.7	1.7	135.7	1.3	134.6	1.0	
132.9	2.7	142.2	3.1	132.9	2.5	134.6	1.6	135.4	1.0	
Ave	142.7	3.0	137.8	2.9	135.1	2.2	136.0	1.5	135.8	1.4
% <i>Il</i>	25.4	21.9		18.4		19.3		19.3		

Final average, 1000 trials, 138 mm. or an illusion of 21%. Mean variation, 2.7%.

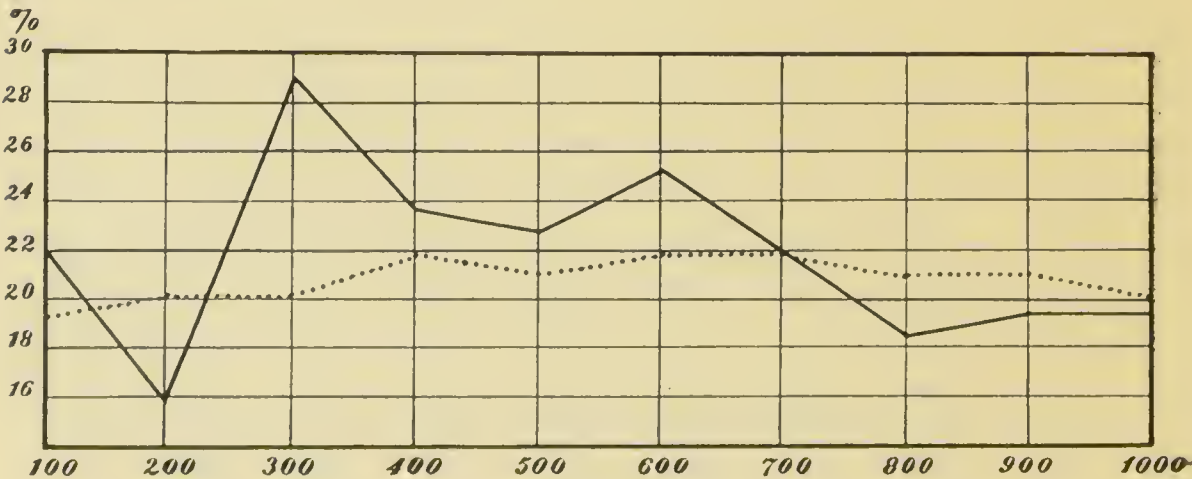


FIGURE 3

TABLE XXIII. *Observer 4*

<i>1st day</i>		<i>2d day</i>		<i>3d day</i>		<i>4th day</i>		<i>5th day</i>		
<i>E</i>	<i>d</i>	<i>E</i>	<i>d</i>	<i>E</i>	<i>d</i>	<i>E</i>	<i>d</i>	<i>E</i>	<i>d</i>	
123.9	2.2	121.2	1.0	121.0	1.6	116.4	2.8	120.2	2.2	
120.6	2.6	125.1	1.9	121.5	2.1	113.3	1.7	117.9	1.7	
121.1	1.5	119.2	1.4	123.9	1.3	114.7	0.9	117.8	1.2	
119.3	1.5	119.0	1.2	122.0	1.0	114.7	1.3	114.6	1.6	
118.1	1.3	120.1	1.3	122.5	1.6	113.8	1.6	115.4	1.4	
118.0	1.0	120.6	1.6	124.7	0.7	113.8	1.1	115.5	1.4	
117.5	1.9	120.7	1.1	124.3	0.9	114.1	0.7	121.3	2.2	
119.3	1.8	120.2	2.2	122.7	1.1	117.0	1.2	117.2	1.0	
120.5	2.1	119.2	1.2	122.7	1.7	118.7	1.7	118.6	1.6	
119.8	1.8	117.3	1.9	123.2	1.8	117.0	1.8	116.7	1.5	
<hr/>		<hr/>		<hr/>		<hr/>		<hr/>		
Ave	119.8	1.8	120.3	1.5	122.9	1.4	115.4	1.5	117.5	1.6
% Il	5.2		5.2		8.0		0.9		3.5	
<hr/>										
<i>6th day</i>		<i>7th day</i>		<i>8th day</i>		<i>9th day</i>		<i>10th day</i>		
<i>E</i>	<i>d</i>	<i>E</i>	<i>d</i>	<i>E</i>	<i>d</i>	<i>E</i>	<i>d</i>	<i>E</i>	<i>d</i>	
119.4	2.0	120.7	1.3	122.0	1.6	123.7	1.1	122.1	1.1	
119.4	1.0	119.3	1.4	122.4	1.0	122.7	1.9	121.8	0.8	
121.0	1.2	119.0	1.3	123.7	0.5	123.7	1.5	123.6	0.8	
121.3	1.9	119.0	2.0	126.1	1.3	123.3	1.5	124.9	1.3	
120.2	1.6	120.4	1.6	124.7	1.9	125.4	2.1	124.4	2.2	
122.5	1.1	120.7	2.3	122.3	0.9	125.5	0.8	126.4	1.6	
121.4	1.8	120.4	3.6	124.5	1.7	124.8	1.4	124.0	1.0	
121.9	2.0	121.2	1.6	123.7	0.9	124.5	1.5	124.5	1.5	
122.1	1.3	120.4	1.0	123.0	1.4	125.9	1.7	124.7	1.5	
119.7	1.1	119.0	0.8	125.0	1.9	126.0	1.4	124.0	1.4	
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Ave	121.0	1.5	120.0	1.7	123.7	1.3	124.6	1.5	124.0	1.3
% Il	6.1		5.2		8.7		9.6		8.7	

Final average, 1000 trials, 121 mm. or an illusion of 6.1%. Mean variation 1.4%.

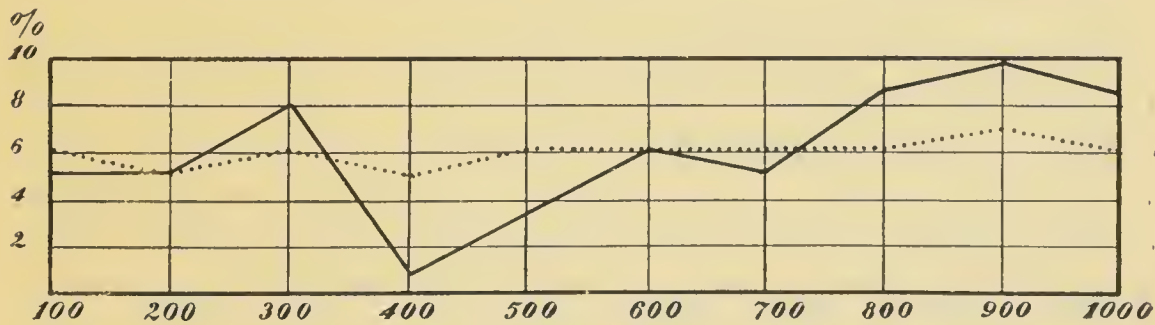


FIGURE 4

hundred trials in each practice period were divided into ten consecutive groups of ten trials each. The averages of these groups are given in Tables XXI, XXII, and XXIII, which represents respectively the judgments of Observers 1, 2, and 4. The results are represented graphically by two curves for each observer in which the numbers on the ordinates represent the percentage of illusion and the numbers on the abscissas represent the hundreds of trials. The continuous line shows the variation for the ten days and the dotted line, which is a composite of the daily curves, shows daily variation. Decrease in the force of the illusion with practice would be indicated by a fall in the curves.

The curves for Observer 1 are found in Fig. 2. The continuous line is somewhat irregular but shows no improvement as the result of practice. The dotted line indicates a slight daily improvement. The total illusion is 21%, with a mean variation of 2%.

Fig. 3 represents the curves for Observer 2. The continuous line is very irregular but it does not show any improvement worth considering. The dotted line shows no constant daily variation. The illusion, as obtained from the thousand trials, is 21%, with a mean variation of 2.7%. The strength of the illusion was very surprising to the observer himself. When shown his records and curves after the completion of the series, he was astonished at the force of the illusion, for he had been under the impression that he "was getting it exactly right."

The strength of the illusion for these two observers is unusual. It is probable that it is due to individual peculiarities, for it is known that the force of an illusion varies with different individuals, and even with the same person at different times, as the irregularities in the curves show. From previous tests it was known that the illusion for Observer 2 was strong (Series III, Observer 4), which is the reason he was chosen. The strength of the illusion for



Observer 1 was not known beforehand but it is extraordinary.

The curves for Observer 4 are given in Fig. 4. The dotted line approaches the straight line and indicates the absence of a constant daily variation. The continuous line, which represents the variation for the ten days, is rather irregular. The first thing noticed is that for the series as a whole there is no improvement as the result of practice, for the illusion at the completion of the test is fully as strong as at the beginning. On the fourth day a marked decrease in the force of the illusion is observed, for which no explanation can be offered other than that of an unconscious allowance. The observer expressly stated that he was not allowing for the illusion. At the close of the sixth period the observer watched while a few trials were made upon a little boy, and remarked that the boy should have made his estimates at least 8% shorter. By accident he saw the boy's records and was much surprised, for if the boy's estimates had been 8% smaller the illusion would not have come out at all. From this, Observer 4 concluded that he himself must have been allowing for the illusion and a rise in the curve for the subsequent tests may be due to a reaction against this. What has just been said does not apply to the first six periods and therefore fails to account for the fall in the curve on the fourth day and its rise on the fifth and sixth days. For Observer 4 the total effect of the illusion of the vertical as obtained from the average of one thousand trials is 6%, with a mean variation of 1.4%. The records with which Observer 4 was especially satisfied and which he pointed out as his best judgments are given in Table XXIV. The average illusion for these special estimates is greater than the average illusion for all the trials.

The following conclusions may be drawn from this series:

The illusion fluctuates in strength from day to day, especially for the observers who are aware of its existence.

TABLE XXIV. *Observer 4*

<i>1st day</i>	<i>2d</i>	<i>3d</i>	<i>4th</i>	<i>5th</i>	<i>6th</i>	<i>7th</i>	<i>8th</i>	<i>9th</i>	<i>10th</i>
<i>E</i>	<i>E</i>	<i>E</i>	<i>E</i>	<i>E</i>	<i>E</i>	<i>E</i>	<i>E</i>	<i>E</i>	<i>E</i>
117	122	124	114	123	121	123	125	124	125
115	120	122	116	117	120	124	123	125	120
120	118	125	117	121	122	119	126	122	123
			122	118	123	118	128	127	122
			121	117	120	116	126	124	124
			116	117	119	123	123	127	124
			116	117	121	115	125	126	127
				118	122	125	127	124	128
				116		120	124	126	127
				115		120	125	127	126
				118		119	124	128	122
				120			126	126	126
				118			126		127
							128		125
									124
Ave 117	120	124	117	118	121	120	125	126	125
% Il 2.6	5.2	8.7	2.6	3.5	6.1	5.2	9.6	10.5	9.6

Final average, 89 trials, 121.8 mm. or an illusion of 7%. Mean variation 2%.

The practice gained in 1000 trials does not decrease the force of the illusion of the vertical for the line: this is equally true of observers who know of the illusion and of those who do not know of it.

For one observer, who has had extensive experience in the observation of this illusion for years, the illusion still has a normal force.

*Series IX*

The purpose of this series was to supplement Series V by measuring the illusion of the vertical in some of the standard forms for which it was not measured in Series V, and incidentally to confirm further the area and volume illusions. The tests were made in February, 1902.

One of the observers (No. 2) was Observer 4 of Series VIII, and the other five were members of the class in elementary psychology. These five observers knew something about illusions and in some cases may have made an unconscious allowance. With reference to all but the area and volume illusions the observers belong to the first type. They knew nothing about the area and volume illusions.

The method of production was used as in Series VIII. To turn from the standard form to the compared line required an excursion of the head of about 30 degrees. The standard forms were placed to the right of the observer, 1 meter away, and at right angles to the line of regard.

The different cases are described in Table XXV. In Table XXVI the signs of the various illusions involved and the percentage of illusion are given. The individual records are found in Table XXVII. Four trials were made on each case by each observer. The term 'altitude' is used with its signification in geometry as a line drawn from the apex perpendicular to the base; the altitude of a form may thus be either vertical or horizontal.

Ten of the cases from Series V were necessarily repeated and a comparative list of these is found in Table XXVIII. The methods of the two series differed in that selection was employed in Series V and production in this series. In both series the observer turned from the standard form to the compared line but in Series IX the error due to the turning is less than in Series V, because the distance between the two forms is less, and accordingly a smaller illusion is to be expected in this series. A glance at the results for the single forms in the two series shows that they agree in general, but that the illusions are a little stronger in Series V, as was expected. The records on the double forms show a distinct disagreement and at present no adequate explanation for this discrepancy can be given, except that there is great uncertainty in the perception of these double forms.



TABLE XXV.

<i>Case</i>	<i>Dimension of Standard Form Measured</i>	<i>Direction</i>	<i>No. in Fig 1</i>
1	Length of line	Vertical	16
2	Length of line	Horizontal	16
3	Altitude of triangle	Vertical	5
4	Altitude of triangle	Horizontal	5
5	Height of plate	Vertical	1
6	Width of plate	Horizontal	1
7	Length of triangle and plate	Vertical	9
8	Length of triangle and plate	Horizontal	9
9	Altitude of cone	Horizontal	7
10	Length of cylinder	Horizontal	3
11	Length of cone and cylinder	Horizontal	11
12	Altitude of pyramid	Horizontal	6
13	Altitude of pyramid	Vertical	6
14	Length of pyramid and cube	Horizontal	10
15	Altitude of cone	Vertical	7
16	Length of cylinder	Vertical	3
17	Length of cone and cylinder	Vertical	11
18	Length of pyramid and cube	Vertical	10
19	Height of cube	Vertical	2
20	Width of cube	Horizontal	2

In each case the compared form was a horizontal line.

Statements of the illusion of the vertical were obtained by comparing the same dimension of a form alternately in the vertical and horizontal positions with a horizontal line and taking the difference between the results of the two comparisons. For instance, when the altitude of the vertical cone is compared with a horizontal line, the illusion of the vertical is involved; but when the altitude of the horizontal cone is compared with the horizontal line, it is not involved. The difference between the two comparisons is plainly a statement of the illusion of the vertical. The other illusions change but little with the change in position of the form.

TABLE XXVI.

<i>Case</i>	<i>V</i>	<i>M-L</i>	<i>A</i>	<i>VI</i>	<i>L</i>	<i>C-L</i>	<i>% Il</i>
1	+	0	0	0	0	0	+11
2	0	0	0	0	0	0	+ 5
3	+	—	+	0	0	0	+ 9
4	0	—	+	0	0	0	+ 5
5	+	0	+	0	0	0	+10
6	0	0	+	0	0	0	+ 7
7	+	—	+	0	—	0	0
8	0	—	+	0	—	0	— 2
9	0	—	+	+	0	+	+11
10	0	0	+	+	0	+	+13
11	0	—	+	+	—	+	+ 2
12	0	—	+	+	0	0	+10
13	+	—	+	+	0	0	+11
14	0	—	+	+	—	0	+ 1
15	+	—	+	+	0	+	+12
16	+	0	+	+	0	+	+19
17	+	—	+	+	—	+	+ 2
18	+	—	+	+	—	0	+ 3
19	+	0	+	+	0	0	+14
20	0	0	+	+	0	0	+11

*V*, illusion of the vertical.

*M-L*, Müller-Lyer illusion.

*A*, area illusion.

*VI*, volume illusion.

*L*, illusion of length.

*C-L*, illusion of cylinder length.

*% Il*, percentage of illusion.

The following statements of the illusion of the vertical were obtained:

Line	6%	Case 1 minus Case 2
Cylinder	6%	Case 16 minus Case 10
Triangle	4%	Case 3 minus Case 4
Cube	3%	Case 19 minus Case 20
Triangle and plate	2%	Case 7 minus Case 8
Pyramid and cube	2%	Case 18 minus Case 14
Cone	1%	Case 15 minus Case 9
Pyramid	1%	Case 13 minus Case 12
Cone and cylinder	0	Case 17 minus Case 11

The conclusion from Series V, that the illusion of the vertical varies with the form, is abundantly supported by

TABLE XXVII.

<i>Obs</i>	<i>2</i>	<i>1</i>	<i>4</i>	<i>3</i>	<i>6</i>	<i>5</i>	<i>Ave</i>			
<i>Case</i>	<i>E d</i>	<i>E d</i>	<i>E d</i>	<i>E d</i>	<i>E d</i>	<i>E d</i>	<i>E d</i>	<i>Il</i>	<i>% Il</i>	<i>% D</i>
1	124 0	137 3	128 5	118 5	130 3	126 14	127 5	+13	11	3
2	116 2	127 4	117 3	115 1	127 2	118 6	120 3	+ 6	5	4
3	125 1	132 2	130 4	116 3	120 2	118 8	124 3	+10	9	5
4	120 1	126 3	130 3	110 4	118 2	118 5	120 3	+ 6	5	4
5	123 1	138 5	135 3	113 1	127 6	121 4	125 3	+11	10	6
6	120 2	128 4	140 5	109 6	122 1	113 4	122 4	+ 8	7	7
7	242 6	253 17	251 10	193 9	207 9	226 4	229 9	+ 1	0	9
8	238 6	249 7	240 6	189 10	202 4	217 6	223 7	— 5	2	10
9	122 3	138 3	132 4	114 2	123 6	129 4	126 4	+12	11	6
10	123 4	128 5	140 4	127 1	125 3	133 4	129 4	+15	13	4
11	242 1	256 13	247 11	209 14	213 8	229 10	233 10	+ 5	2	7
12	121 2	135 3	141 3	113 5	127 4	115 6	125 4	+11	10	10
13	124 3	132 4	147 2	114 2	123 8	118 4	126 4	+12	11	8
14	247 5	250 8	268 3	204 10	198 3	221 9	231 6	+ 3	1	11
15	125 4	137 3	140 2	113 4	128 7	126 9	128 5	+14	12	6
16	130 3	140 6	154 3	121 2	138 4	134 4	136 4	+22	19	7
17	248 6	242 2	266 8	195 6	199 7	242 11	232 8	+ 4	2	10
18	154 5	246 7	271 1	192 2	192 11	251 11	234 3	+ 6	3	12
19	128 1	134 4	143 2	123 4	137 1	115 2	130 2	+16	14	7
20	120 3	128 3	149 4	118 4	132 3	110 6	126 4	+12	11	9

The notation is the same as in Table I.

the present series. Although it is difficult to group the forms in the order of their complexity, the general statement may be made that the illusion is smaller for the more complex forms, such as the cone and cylinder (Form 11), than for the simple forms, such as the line.

The presence of the area and volume illusions in this series is clearly indicated by the results. When the width of the plate is compared with the line (Case 6), the illusion of area is 7%. The differences between the results for the plate and cube give statements for the illusion of volume for the cube: 4% for both height and width (Case 19 minus Case 5, and Case 20 minus Case 6). The illusion of area for the triangle amounts to 5% plus the Müller-Lyer effect (Case 4). The volume illusion for the cone and the pyramid amounts to 6% and 5% respectively,



TABLE XXVIII.

<i>Dimension of Standard Form Measured</i>	<i>Series V</i>		<i>Series IX</i>	
	<i>Case</i>	<i>% Il</i>	<i>Case</i>	<i>% Il</i>
Length of line	5	11	1	11
Altitude of cone	6	16	15	12
Altitude of pyramid	8	13	13	11
Altitude of triangle	10	11	3	9
Length of cylinder	15	20	16	19
Height of cube	16	17	19	14
Height of plate	17	15	5	10
Length of cone and cylinder	12	11	17	2
Length of pyramid and cube	13	12	18	3
Length of triangle and plate	14	11	7	0

In this table the signs are all plus.

plus the Müller-Lyer illusion (Case 9 minus Case 4 for the cone, and Case 12 minus Case 4 for the pyramid). In the triangle and plate combined, the area illusion and the illusion of length are outweighed by the Müller-Lyer illusion by 2% (Case 8). The volume illusion for the cone and cylinder in combination is 4% (Case 11 minus Case 8) and for the pyramid and cube it is 3% (Case 14 minus Case 8).

The following points are brought out in this series:

For the single forms there is on the whole a very satisfactory agreement between this series (method of production) and Series V (method of selection).

The illusion of the vertical for the line, cone, pyramid, triangle, cylinder, cone and cylinder, pyramid and cube, and triangle and plate, is again found to vary with the form.

The area and volume illusions are further demonstrated.

The evidences of this series, together with those of Series V, indicate that the illusion of cylinder length is present to a slight degree in the cone.

*Series X*

The purpose of this series was to determine the variation of the illusion of the vertical with the size of the square and with the length of the line. Up to this point the chronological order has been followed in the discussion of the various series of experiments. This series, however, preceded all the others in point of time, the records being dated November, 1899. Two sets of experiments are grouped together to form this series.

The first set was made to determine the variation of the illusion of the vertical with the size of the square. The method of production was used. A large sheet of cardboard, tinted a light pink, was placed before the observer at right angles to the line of regard. In front of this was a strip of white cardboard, the width of which was the standard distance. A strip of the pink cardboard of the same width was placed over this white strip, leaving only so much of it exposed as the observer judged to be a square. The experimenter moved this pink strip as the observer, who sat 1 meter away, directed. The standards were 38, 57, 114, 228, and 456 mm.

The observers were two university professors and five members of a class in experimental psychology, all belonging to the first type. They all knew of the illusion that was being studied, but upon request they tried not to make conscious correction for it. Forty determinations were made by each observer upon the vertical position of the standard with the horizontal dimension varying (A), and forty determinations upon the horizontal position of the standard with the vertical dimension varying (B). The results for these two positions of the standard are given in percentages in Table XXIX, A and B respectively.

The illusion of the vertical is the only one that enters. With the standard vertical, it causes the width of the form to be made too large; with the standard horizontal, it

TABLE XXIX. (A).

<i>Standard</i>	38		57		114		228		456	
<i>Obs</i>	% <i>Il</i>	<i>d</i>	% <i>Il</i>	<i>d</i>	% <i>Il</i>	<i>d</i>	% <i>Il</i>	<i>d</i>	% <i>Il</i>	<i>d</i>
2	+ 8	3	+ 4	3	+ 4	2	+ 6	1	+ 2	2
1	+ 4	4	+ 1	4	0	3	+ 1	3	+ 4	3
4	+ 1	2	— 2	5	+ 2	4	+ 7	3	+ 4	3
6	+ 5	2	+ 4	2	+ 4	2	+ 2	2	+ 2	2
8	+ 8	3	+ 5	1	+ 7	2	+ 4	3	+ 1	2
10	+13	3	+ 7	4	+11	3	+ 7	3	+ 4	2
3	+ 6	3	0	1	+ 1	2	+ 1	1	+ 1	1
	—	—	—	—	—	—	—	—	—	—
Ave	+ 6	3	+ 3	3	+ 4	3	+ 4	2	+ 3	2

TABLE XXIX. (B).

<i>Standard</i>	38		57		114		228		456	
<i>Obs</i>	% <i>Il</i>	<i>d</i>	% <i>Il</i>	<i>d</i>	% <i>Il</i>	<i>d</i>	% <i>Il</i>	<i>d</i>	% <i>Il</i>	<i>d</i>
2	— 8	2	— 7	2	— 4	2	— 6	6	— 2	2
1	— 1	2	— 4	3	0	2	0	2	0	4
4	— 3	3	— 7	2	+ 2	4	+ 4	4	— 2	3
6	— 3	3	— 2	3	— 1	1	0	1	— 1	1
8	— 6	0	— 5	0	— 4	1	— 5	1	— 8	1
10	+ 3	5	+ 2	2	0	2	— 2	2	— 3	1
3	+ 6	3	+ 2	1	+ 3	2	0	3	— 4	2
	—	—	—	—	—	—	—	—	—	—
Ave	— 2	3	— 3	2	— 1	2	— 1	3	— 3	2

causes the height of the form to be made too small. That is, there should be a plus sign and a minus sign respectively for the vertical and horizontal positions of the standard.

The results reveal no definite constant tendency for the illusion of the vertical to vary with the size of the square. The observers reacted against the illusion wherever there is a minus sign in Table XXIX (A), and a plus sign in Table XXIX (B), and probably they reacted to a less extent in other instances.

The second set of experiments in this series was made at about the same time and its purpose was to determine the variation of the illusion of the vertical with the length of the line. The method of production as employed in



TABLE XXX.

<i>Standard</i>	<i>38</i>			<i>76</i>			<i>114</i>			<i>152</i>			<i>190</i>		
<i>Obs</i>	<i>% Il</i>	<i>d</i>		<i>% Il</i>	<i>d</i>		<i>% Il</i>	<i>d</i>		<i>% Il</i>	<i>d</i>		<i>% Il</i>	<i>d</i>	
2	—	3	4	+	5	1	+	5	4	+	11	5	+	11	8
4	—	1	1	+	7	4	+	9	10	+	16	11	+	23	15
6	—	1	1	—	1	2	—	5	5	—	8	7	—	9	5
8	+	2	2	+	5	3	+	4	1	+	5	4	+	3	6
12	+	2	1	+	7	3	+	8	2	+	15	4	+	16	9
10		0	1	+	4	4	+	12	9	+	13	9	+	18	7
	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
Ave	0	2		+	5	4	+	5	5	+	9	7	+	10	8

Case 1 of Series V was used. The standard lines were 38, 76, 114, 152, and 190 mm. in length and were in the vertical position. Twenty judgments were made upon each line by each of five observers and two judgments by each of ten other observers, all of whom belong to the first type. The records of the ten observers are grouped together and designated by the number 10. The records are summarized in Table XXX, where the percentage of illusion and the mean variation in millimeters are given. The results indicate that the illusion varies with the length of the line; for the five standard lines the illusion is respectively 0, 5%, 5%, 9%, and 10%.

It may be concluded that, for the observers in this series who tried the experiments upon the square, the illusion of the vertical does not vary constantly with the size of the square; but for those who tried the experiments upon the line, the illusion of the vertical shows a distinct tendency to increase in force with the increase in the length of the line.

### *Series XI*

In this series the results of some experiments upon common objects are reported. The illusions which have been studied in the preceding series are not such as are found

only in a psychological laboratory; they are present in such forms as the book, box, hat, cup, barrel, coffee-pot, cupboard, churn, jug, basket, kettle, house, tree,—in fact in almost any object with which one has experience. For instance, it is well known that the height of the crown of an ordinary silk hat is greatly overestimated. This is usually attributed to the illusion of the vertical alone, but in the light of the present studies this accounts for only a part of the total illusion. The illusions in the silk hat are the same in kind as those in the vertical cylinder; the height of the hat is overestimated on account of the illusion of the vertical and the illusion of cylinder length, and the size of the hat as a whole is overestimated on account of the area and volume illusions. Experiments were planned to measure the illusions in some of these common forms, but for want of time they were not carried out. However, two excursions were made during which several large objects in the vicinity of the University campus were studied.

There were three observers in the first excursion and nine in the second; they will be referred to as the 3 Obs. and the 9 Obs., respectively. The 3 Obs., two instructors and an advanced student, were familiar with all the illusions described and therefore belong wholly to the first type.<sup>1</sup> The 9 Obs. were young men, members of the class in elementary psychology; with regard to the illusion of the vertical they belong to the first type, but with regard to the other illusions they belong to the second type. The two sets of observers did not all try the same tests.

In making the experiments four different methods were employed:

By the first method the required magnitude was marked off on a surveyor's steel tape. If the height of a building

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<sup>1</sup> None of these observers knew anything in regard to whether the illusion would appear in the large objects, and the objects were not measured to determine their true sizes until after the first excursion.

was being studied, the observer was stationed at a distance from the building equal to the height. One end of the tape was fastened at the corner of the building a short distance above the ground, and an assistant held the other end so that the tape was stretched outward at right angles to the observer; the experimenter moved a pointer over the tape as the observer directed and thus marked off the distance from the corner and the number of feet was then read off from the tape. This method corresponds roughly to the method of production in the regular tests and for large objects it is sufficiently accurate.

In the second method the observer stationed himself at such a distance from the object as he considered equal to its height. This method was not very accurate and was used only to supplement the other methods.

The third method was the estimation of the ratio of the dimensions of the object. If for instance, the side of a building was being studied, the observer was asked: "If the height consists of ten units, how many such units are there in the length?"

The fourth method was to indicate upon the building a horizontal distance equal to the vertical; that is, the observer marked off a square. The experimenter stood close to the front of the building and moved a vertical pointer to the right or left as the observer directed until a distance was marked off which was judged to be equal to the height of the building.

The first object studied was the Home Education building. This is a large brick building with plain front. The stone foundation extends about four feet above the ground level and on this are four courses of red brick. From this to the eaves the brick is painted a dull gray. Counting from the top of the line of red brick to the eaves, the building is 32 feet high. This was taken as the standard distance. The length of the building is 60 feet.

Height, by the first method: The illusion of the vertical,



the area illusion and the volume illusion all tend to cause an overestimation. The 3 Obs. overestimated the height by 31%, (%D, 14); and the 9 Obs. overestimated by 34%, (%D, 8).

Height, by the second method: The same illusions are involved as with the first method but there are sources of error in the method. The 9 Obs. stood too far away by 69%, (%D, 16).

Height, by the fourth method: The illusion of the vertical is the only one entering in this instance. The 9 Obs. made the square too wide by 22%, (%D, 9).

Length, by the first method: The area and volume illusions tend to cause an overestimation. The result for the 3 Obs. is zero, (%D, 5); the result for the 9 Obs. is +18%, (%D, 8). The 3 Obs. probably made allowance for the illusions, as they knew of them, and the 9 Obs. did not.

Ratio of height to length, by the third method: According to the illusion of the vertical the height is overestimated and the units would be large, therefore a smaller number of them would be contained in the length; that is, the sign should be minus. The result for the 9 Obs. is -16%, (%D, 6).

The west side of the Physics Hall was also studied. This is a plain red brick building with a stone foundation. The height above the water-table is 43.5 feet and the length is 89 feet. The measurements upon it should be compared with the corresponding ones upon the Home Education building.

Height, by the first method: The effect of the illusion of the vertical, the area illusion and the volume illusion amounts to 18% for the 9 Obs.

Height, by the second method: In addition to large sources of error due to the method, the same illusions are involved as with the first method. The 9 Obs. stood too far away by 53%, (%D, 5).

Height, by the fourth method: The illusion of the ver-

tical caused the 9 Obs. to make the square 16% too wide, (%D, 11).

Ratio of height to length, third method: The illusion of the vertical enters with a minus sign. The result for the 9 Obs. is —19%. Five students in the department of Civil Engineering, belonging to the second type of observers, were asked to try this test. The result for them is —23%.

The excursion next proceeded to the smoke stack of the University heating plant. The chimney proper is octagonal and is built of buff pressed brick. It rests upon a square stone foundation which extends 20 feet above the ground level. The brick work above this is 103 feet, making the total height as used for the standard, 123 feet. A good unobstructed view of it was obtained from across the street, west.

Height, by the first method: The illusion of the vertical, the area and volume illusions and possibly the illusion of cylinder length enter. The 3 Obs. overestimated by 3%, (%D, 8); and the 9 Obs. by 15%, (%D, 9).

Height, by the second method: The same illusions enter as in the first method. The 3 Obs. stood back too far by 16%, (%D, 12); and the 9 Obs. by 28%, (%D, 12).

An isolated telephone pole, 31 feet high, was also measured. According to the first method the illusion of the vertical for the height of the pole for the 9 Obs. was 26%, (%D, 13).

The illusions in two large painted signs were measured by the third method. Both the signs, which were painted on the side of a two-story building, were taller than they were wide, the ratios being 19:10 and 12:10. In estimating the ratio of the width to the height, the height should be overestimated according to both the illusion of length and the illusion of the vertical. The results for the 9 Obs. were 21%, (%D, 10) and 17%, (%D, 3) for the two signs, respectively.

A very beautiful and symmetrical hard maple tree, standing somewhat isolated upon the campus, was also studied.



It had a distinct conical shape and was in full leaf. From the ground to the apex the tree measured 60 feet, and had a broadly spreading base. Just what illusions are involved in the perception of a tree has not been determined. With this particular tree the Müller-Lyer effect certainly was present (—), also the illusion of the vertical (+), the illusion of cylinder length (+), and the area and volume illusions (+). When the 9 Obs. compared the height of the tree with the tape according to the first method, they underestimated its real height by 13%, (%D, 5). These results were so striking that further experiments were made with seventeen observers, all but three of whom belonged to the second type. The results agree with those obtained above. When the height of the tree is estimated by the second method it is judged to be taller than by the first method. For the 17 Obs. the height of the tree is underestimated by 13%, (%D, 8); according to the second method it is underestimated by 3%, (%D, 9). This difference between the two methods is in accord with the differences found for the buildings and the smokestack. At various times different persons were asked to guess the height of the tree but none of them ever estimated its true height, the judgments as a rule being made too small by several feet. What motive for illusion can there be in the perception of the tree that is strong enough, in coöperation with the Müller-Lyer illusion, to counterbalance the combined effect of the illusion of the vertical, the illusion of cylinder length, the area illusion and the volume illusion? Further experiments will be made to determine this and how much may be due to the method of comparison. It seems to be a matter of common experience with those who have seen trees felled, that the tree looks much taller when standing upright than when lying upon the ground. It may be that an opinion to that effect may have led the observers to make semi-conscious or unconscious correction for it.

The stand-pipe at the C., R. I. & P. R. R. station is a



tall metal cylinder 52 feet high and 12.5 feet in diameter. It rests directly upon a concrete foundation 1 foot in height above the ground. No tall objects are near it.

Height, by the first method: The illusion of the vertical, the illusion of cylinder length, the area illusion and the volume illusion should cause an overestimation; but the result for the 3 Obs. is  $-9\%$ , ( $\%D$ , 4).

Diameter, by the first method: The area and volume illusions should cause an overestimation. The 3 Obs. overestimated by  $12\%$ , ( $\%D$ , 0).

Ratio of diameter to height, by the third method: According to the illusion of the vertical and the illusion of cylinder length the result should have a plus sign. For the 3 Obs. the result is  $-9\%$ , ( $\%D$ , 15).

These results are confusing; the test must be repeated upon observers who are not aware of the illusions.

Measurements were also made upon two cylindrical oil tanks. The first tank was 20 feet in both height and diameter and had a flat roof. It corresponded to the equal cylinder in the regular experiments. The 3 Obs. were the only ones who tried the tests upon the tanks.

Height, by the first method: The illusion of the vertical, the illusion of cylinder length, the area illusion and the volume illusion all tend to cause an overestimation. The 3 Obs. overestimated by  $15\%$ , ( $\%D$ , 15).

Diameter, by the first method: The illusions of area and volume enter with plus signs. The result is  $+10\%$ , ( $\%D$ , 10).

Ratio of diameter to height, by the third method: The illusion of the vertical and the illusion of cylinder length tend to cause an overestimation. The illusion is  $20\%$ , ( $\%D$ , 10).

The second tank was 16 feet high and 10.5 feet in diameter.

Height, by the first method: The combined effect of the illusion of the vertical, of cylinder length, of area and of volume is  $+13\%$ , ( $\%D$ , 19).

Diameter, by the first method: The area and volume illusions should cause an overestimation of the diameter. The diameter was overestimated by 10%, (%D, 10).

Ratio of diameter to height, by the third method: The effect of the illusion of the vertical and the illusion of cylinder length is +19%, (%D, 10).

There is great variation in these tests upon common objects. This is to be accounted for by the difference in the amount of knowledge possessed by the observer, by the individual peculiarities, the great uncertainty in the perception of the large forms, the complexity of the conditions, and the relative crudeness of the methods employed. The experiments are merely suggestive; they are too fragmentary to warrant any general conclusions, but in regard to the perception of a side of a building, two facts seem to be demonstrated:

The illusion of the vertical, the area illusion, and the volume illusion are all present.

The illusion of the vertical is greater for these large objects than for the small objects.

### *Critical Remarks*

In the description just given of the eleven series of experiments the primary aim has been to present the full data to the reader and to place in the hands of future investigators records the significance of which has by no means been exhausted by the formal conclusions presented in this article. The writer has simply furnished the complete data and stated her conclusions as tersely as possible without elaborate discussion; the reader is referred to the tables of results for mean variations and all other such factors as enter into the interpretation of the records.

The discussions and conclusions have been based almost entirely upon the averages of the results in each series;

very little attention has been given to the records of the individual observers. A great deal might have been gained by a study of the individual observers; as for instance, a determination of the variation of the illusion with temperament and physical condition, the relative strength of the different illusions for the same person, their consistency under different conditions, variations with sex and mental ability, etc., but all this was prohibited by the closely defined limits of this report. The individual records are given in full and they may be reinterpreted at any time with these problems in view. The records also show the prevalence or constancy of the illusions for the different observers.

In many instances account has been taken of a small percentage of illusion where the corresponding mean variation has been rather large. This has been done only in those cases where there has been a clear motive for the appearance of the illusion and the variation has been normal for the particular method employed.

An apology must be given for a very rigid manipulation of the figures. In the rather intricate discussions of the results it was very desirable, for the sake of clearness, to make definite and, for the most part, unqualified statements. The fact was fully kept in mind that the results were only of relative value. The writer also regrets a certain inconsistency in the use of fractions. The apology is not that fractions were omitted in some instances, but that fractions were used at all. In the effort to interpret the results correctly this inconsistency was overlooked until the advantage to be gained from its correction would not justify the necessary expenditure of time.

Records of observers belonging to different types have in some instances been grouped together. Much confusion would have resulted from an attempt to keep the records of the different types separate. The observers as a class are representative of university undergraduates, chiefly



juniors and seniors. The individual observers have, however, been described with reference to their respective types.

With one exception, no attempt was made to determine the condition of the eyes of the observers, but no experiments were made upon any one whose eyes were obviously in a weak or strained condition. It is possible that some of the observers were troubled with errors of refraction in the eye and that the results were influenced by these defects.

The term illusion has been used in two ways: first, a motive, as when the conflict or struggle of illusions was spoken of; and second, an error, as the amount of illusion. The use of the word motive is self-evident.

### *General Summary*

The following illusions have been considered more or less fully in the foregoing discussions: the illusion of the vertical, the area illusion, the volume illusion, the illusion of cylinder length, the Müller-Lyer illusion, and two illusions due to contiguity. Of these the Müller-Lyer illusion and the illusion of the vertical are well known. The illusion of length was reported in the article preceding this, which is really a part of the present research. The area illusion, the volume illusion, and the illusion of cylinder length have not, to the writer's knowledge, been reported before. The aim has been to make a detailed study of the illusion of the vertical, and to determine what other illusions enter into the perception of the size and form of the principal types of common objects.

A full account of the illusion of the vertical would include a discussion of the following general features: the prevalence and average strength of the illusion; variation with the form of lines and two and three dimensional ob-

jects; variation with size; variation with the distance from the observer; variations due to the angle at which the objects are seen; variations due to complications with other illusions; variation with the different standard methods of measuring illusions; variation with the age of the observers, and also with sex, intelligence, attention, environment, experience or familiarity, knowledge, practice, etc.; a complete discussion with crucial tests of the current theories to explain the illusion; and miscellaneous features, such as the duration of the stimulus, indirect vision, etc. The facts that have been determined in the present research will be summarized briefly.

The range of the illusion of the vertical is large; it varies for different observers from less than 1% to over 20%. The average illusion for the vertical line among adult observers who knew of it and were careful is about 6%. No case in which the illusion of the vertical did not appear was found which could not be accounted for by a reaction against the illusion or carelessness on the part of the observer. Very small percentages of illusion are usually to be explained in the same manner.

The illusion of the vertical varies with the form, being stronger for the less complex forms. The simplest form, the vertical line, shows the greatest illusion. The illusion is not so strong in two dimensional objects as in the line, and in three dimensional objects it is still weaker. It is probable, however, that this variation is, at least in part, only apparent; for in the complex forms there are many conflicting motives to illusion present and in their struggle for supremacy much of the manifest illusion of the vertical may be lost. It has also been pointed out in the preceding paper (page 32), that the effect of one illusion partly satisfies the motive of another. In the circle and related forms the illusion is, as a rule, practically absent on account of the knowledge of the geometrical relations of the forms. Under certain con-

ditions of attention, however, the illusion does appear even in the circle.

The illusion of the vertical varies with the size of the object. It increases in force with the increase in the length of the line; the same probably is true for two dimensional forms, although the experiments indicate no positive tendencies. The illusion is greater for the large buildings than for the small objects.

With reference to the variation of the illusion of the vertical with distance, the records show a decrease in the force of the illusion with the increase in distance. This is in accordance with the statement above that the illusion increases with increase in size; in other words, it is stronger for larger and nearer objects.

The angle at which the square is seen does not affect the illusion of the vertical in small objects.

After the errors incident to the methods have been eliminated, the illusion of the vertical is of about the same force for the different methods used.

Several subjective factors affect the illusion of the vertical. The illusion in the square is stronger for children (third type), than for adults (second type); that is, the illusion varies with age. This is due not so much to an undeveloped state of mind as to the fact that the children give unprejudiced judgments. The illusion was found not to vary with intelligence. With regard to the variation with sex there is practically no difference for the boys and girls in the force of the illusion, but among adults the women are less constant than the men. The illusion also varies with the direction of the attention. There are three ways in which the result may be influenced by the direction of the attention: the attention may be directed to the illusion itself; according to its nature, the motive of the illusion may be strengthened by attending to it or not attending to it; and the attention to one dimension of an object changes the apparent proportion of the other dimensions.



There is a greater incentive to make a correction for an illusion when the attention is strongly directed to it. The illusion of the vertical also varies with the knowledge of the illusion. It persists for some persons who have studied it for years, although they may have learned to make proper allowance for it. The illusion decreases in force when it is pointed out; it is stronger and more constant for the naive observers. The illusion was found not to decrease in force with the practice gained from making one thousand judgments. There is a very strong tendency to react against a known illusion. This illusion and other illusions also will lose much of their constancy when they become popularly known, just as the fixed customs of the savage dissolve under civilization.

A complete analysis of the area illusion would correspond somewhat closely to the one given for the illusion of the vertical. Only a few conditions under which this illusion varies have been determined. The experiments indicate that it is present in all objects having area, and its effect is to increase the apparent size of the object.<sup>1</sup> It is of about the same force for all the forms the areas of which are approximately equal. With the exception of two, none of the observers knew of the illusion and accordingly no conscious or unconscious correction was made for it.

In addition to the area illusion, the volume illusion appears in all forms in which the volume is either real or suggested. No constant variation with the different forms was found. As in the case of the area illusion, the observers knew nothing of the volume illusion and introduced no correction for it.

The initial purpose of this research was to study the illusion which has been termed the illusion of cylinder

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<sup>1</sup> The existence of the area illusion may account for the fact that a line between two points looks shorter than the open distance. All the measurements upon areas in terms of lines, where the area illusion has not been eliminated, are vitiated by the existence of the area illusion.

length. The presence of this illusion of overestimation in the length of the cylinder has been fully demonstrated, but no adequate explanation for it has as yet been found. It is distinct from the illusion of the vertical and it is stronger than this illusion for adults. It is not due to one particular method, for it is brought out by the different methods used. It is not limited to the real cylinder, for it appears in the drawing of the cylinder. Neither is it due to a relative underestimation of the diameter, for it is present with its characteristic force in the tests where the ratio of the length and diameter is not considered. But these limitations do not determine the exact explanation of the illusion. The motive for eye-movement in the direction of the length of the cylinder is peculiar and seems to constitute an adequate motive for the illusion; but it has not been demonstrated by the experiments. There is an illusion in the length of the drawn cylinder (in which this motive for eye-movement is absent) equal to that in the real cylinder, but the two results are probably due to radically different causes: in the drawn cylinder it is manifestly a case of confluence due to the presence of the ellipse at the end. Association theories have been considered but it does not seem that they possess as high a degree of probability as the physiological theory. In Series I it was pointed out that this illusion varies with intelligence, being strongest for the dullest pupils; and in Series II it was shown that it did not vary with age. These two statements are in agreement, for an increase in age really means an increase in intelligence; the brightest children were probably as intelligent as the adults.

No direct determinations of the illusion of length were made, although it appeared several times in combination with other illusions. The incidental evidence which was gained supports the former report.

The Müller-Lyer illusion has figured prominently in the discussion of the results. It entered as a complicating factor

TABLE XXXI.

<i>No. in Fig. 1</i>	<i>Standard Form</i>	<i>Dimension of Standard Form</i>							
		<i>Measured</i>	<i>Direction</i>	<i>A</i>	<i>VI</i>	<i>C-L</i>	<i>M-L</i>	<i>V</i>	<i>L</i>
1	Plate (square)	Height	Vertical	+	0	0	0	+	0
1	Plate (square)	Width	Horizontal	+	0	0	0	0	0
2	Cube	Height	Vertical	+	+	0	0	+	0
2	Cube	Width	Horizontal	+	+	0	0	0	0
3	Cylinder	Length	Vertical	+	+	+	0	+	0
3	Cylinder	Length	Horizontal	+	+	+	0	0	0
3	Cylinder	Diameter	Vertical	+	+	0	0	+	0
3	Cylinder	Diameter	Horizontal	+	+	0	0	0	0
4	Sphere	Diameter	Vertical	+	+	0	—?	+	0
4	Sphere	Diameter	Horizontal	+	+	0	—?	0	0
5	Triangle	Altitude	Vertical	+	0	0	—	+	0
5	Triangle	Altitude	Horizontal	+	0	0	—	0	0
6	Pyramid	Altitude	Vertical	+	+	0	—	+	0
6	Pyramid	Altitude	Horizontal	+	+	0	—	0	0
7	Cone	Altitude	Vertical	+	+	+	—	+	0
7	Cone	Altitude	Horizontal	+	+	+	—	0	0
8	Disk	Diameter	Vertical	+	0	0	—?	+	0
8	Disk	Diameter	Horizontal	+	0	0	—?	0	0
9	Triangle and plate	Length	Vertical	+	0	0	—	+	+
9	Triangle and plate	Length	Horizontal	+	0	0	—	0	+
10	Pyramid and cube	Length	Vertical	+	+	0	—	+	+
10	Pyramid and cube	Length	Horizontal	+	+	0	—	0	+
11	Cone and cylinder	Length	Vertical	+	+	+	—	+	+
11	Cone and cylinder	Length	Horizontal	+	+	+	—	0	+
12	Circle	Diameter	Vertical	—	0	0	—?	+	0
12	Circle	Diameter	Horizontal	+	0	0	—?	0	0
13	Drawn square	Height	Vertical	+	0	0	0	+	0
13	Drawn square	Width	Horizontal	+	0	0	0	0	0
14	Ellipse	Long axis	Vertical	+	0	0	—	+	0
14	Ellipse	Long axis	Horizontal	+	0	0	—	0	0
15	Drawn cylinder	Length	Vertical	+	+	+	?	0	+
15	Drawn cylinder	Length	Horizontal	+	+	+	?	0	0
15	Drawn cylinder	Diameter	Vertical	+	+	0	0	+	0
15	Drawn cylinder	Diameter	Horizontal	+	+	0	0	0	0
16	Line	Length	Vertical	0	0	0	0	+	0
16	Line	Length	Horizontal	0	0	0	0	0	0

The notation is the same as in Table XV.



into many of the forms studied, therefore it was necessary to interpret the results with reference to this illusion.

Two illusions due to contiguity in space were brought out by the results. The first has been referred to as an error due to the use of a series of lines; the second, an error of a similar nature, due to the use of a series of plates. The effect of both was eliminated from the results.

In the way of a final summary, an analysis of the illusions involved in the sixteen forms studied is given in Table XXXI. The direction of the illusions is represented by the plus and the minus signs. The illusions are stated in terms of the horizontal line, for the method of production. The force of the illusions would vary with the numerous different conditions, but the direction of the illusions would remain as indicated in the table.

It gives the writer pleasure to acknowledge her obligation to all those who kindly served as observers, and more especially to Dr. C. E. Seashore who most generously gave his time and thought to the promotion of the research, with regard both to the planning of the tests and also to the arrangement of them in final form for presentation.

# THE NEW PSYCHOLOGICAL LABORATORY OF THE UNIVERSITY OF IOWA

BY

GEORGE T. W. PATRICK

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The new Psychological Laboratory of the University of Iowa was completed and occupied in September, 1901. It is situated on the second floor of the north wing of the Hall of Liberal Arts, a new Bedford-stone structure begun in 1897. This is a fire-proof building, constructed at a cost of about \$203,000, 210 feet by 120 feet, and three stories high with a basement. It is heated and ventilated by the fan system combined with direct radiation, a Sturtevant thermostat in each room maintaining the temperature at any desired degree of heat.

Previous to the drawing of the plans of the building, 4500 square feet of clear floor space were set aside for the department of philosophy and psychology, including the laboratory. Through the kindness of the Board of Regents, *carte blanche* was given to the department in conference with the architect in respect to the plans, including all details of construction and arrangement. The building committee of the Board generously granted every reasonable request to the end of securing in the outcome as perfect a laboratory as could be designed within the allotted space.

The rooms with their names and sizes are as follows:

Lecture Room.....	27'— 8"	×	29'—10"
Apparatus Room .....	15'— 7"	×	23'— 2"
Work Shop .....	20'— 0"	×	23'— 5"
Measuring Room .....	15'— 5"	×	18'— 8"
Observing Room .....	16'— 2"	×	12'— 7"

Private Laboratory . . . . .	12' — 3"	×	20' — 0"
Research Room . . . . .	16' — 2"	×	20' — 0"
Research Room . . . . .	19' — 6"	×	20' — 0"
Small Lecture Room* . . . . .	16' — 6"	×	23' — 9"
Library and Seminary Room* . . . . .	20' — 9"	×	26' — 0"
Office . . . . .	10' — 2"	×	20' — 9"
Office . . . . .	9' — 10"	×	23' — 2"

The arrangement of the rooms is such as secures both quiet and convenience. None of the laboratory rooms open directly upon the main hall of the building. Every room is as free as could be desired from noise and disturbance both from within and without the building, while for the two research rooms, still further seclusion is obtained by their location upon the floor above the main laboratory, from which they are reached by a private stairway from the work shop. The rooms are all thirteen feet in height, excepting the observing room which is 10 feet 2 inches in height. The floors are of maple and the other finishing is all of oak. The work shop is wainscoted in oak to the height of six feet and ceiled with oak in such a manner that, when desired, shafting, pulleys, or other fixtures may be attached for the operation of machinery. All the rooms are supplied with gas and electrical lighting. The large lecture room and the work shop are supplied with hot and cold water. The windows of all the rooms except the two offices have extra darkening shades working in grooves at the sides. These darkening shades are hung on heavy rollers boxed into the casing at the top of the window.

A switch-board is built into the wall of the work shop adjoining the battery closet. It is 2 feet by 3 feet in size,

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\*Owing to the recent fires, destroying two of the University buildings, these two rooms are temporarily used by another department. Meanwhile the apparatus room, which adjoins the large lecture room, is used as the library and seminary room, and one of the offices is used as apparatus room.



made of white marble set in an oak frame and contains eighty single terminals, all fused. From the switch-board electrical connections are made by concealed wires running through iron tubing in the walls and under the floors to the lecture rooms and each of the laboratory rooms, providing four circuits to each room except the observing room, which has five circuits, and the battery closet, which has six circuits. The mains from the University power-plant are also brought into the system. The switch-board is used not only for supplying the current to all the rooms but also for bringing different suites of rooms into telephonic or other electrical connection. The terminal boards in the several rooms are found at convenient places in the walls.

The construction of the observing room deserves especial mention. To make a dark room impervious to external light is a matter presenting no serious difficulty. To make a room impervious to external sound or wholly free from the jarring from surrounding rooms or adjacent streets is a problem which has not yet been solved and of course never will be. We made the attempt to approach a little nearer to this end than has hitherto been done. The result is a room as free from external disturbances as is needed in any experiments in which it is necessary to control visual, auditory and tactual stimuli. So far as this has been accomplished, the credit is largely due to the architects, Messrs. Proudfoot and Bird, who worked out many of the details of construction. The position of the observing room is central, occupying a place not otherwise desirable from lack of light. The room rests on an independent foundation, having no solid connection with the rest of the building either below, above, or on the sides. The superstructure which supports the room rests upon a sand bed and a second sand bed at a higher level still further assists in eliminating possible jarring or sound which might be communicated from the ground. The

walls of the room itself, inside the main partitions, which separate the whole space from the surrounding apartments, are made of two four inch walls of hollow tiles separated by an air space and each covered with a thick insulating material made of sea-weed. Inside of all, the walls are plastered and then lined throughout with black broadcloth. The inside room is divided into the main observing room,  $12'-2" \times 12'-7"$ , and a vestibule,  $4'-0" \times 12'-7"$ . The room is entered through five doors, the outer one being an ordinary oak door and the other four specially constructed tight fitting cedar doors, covered with black cloth on the sides and edges. The doors close with strong springs and are held open by automatic catches. The floor is made of Tennessee red cedar and covered with linoleum painted black. The room is heated and ventilated by means of hot air introduced not directly but from the attic through cedar shafts provided with overlapping cloth partitions which admit the air but help to exclude sound. For very fine experiments the ventilating shaft may be closed and the room ventilated during intermissions. The room may be lighted by gas or electricity, the former being introduced through rubber tubes. The furnishing consists of black tables and chairs and a telephone connected with the measuring room. In the exclusion of sound and vibration, as well as in other respects, the room has proved to be a complete success. The loudest stentorian shouting just outside the doors is absolutely unheard within.

The work shop is supplied with a substantial work bench inclosed below for the reception of lumber; also with lathe, tool case and mimeograph, drawing materials, motors, etc., and material cases with glass doors. The measuring room is supplied with a special instrument bench. The other rooms are furnished with suitable tables, chairs, apparatus cases, a students' locker, a chart case, drawing and apparatus tables.

The library and seminary room contains the departmental library.

The equipment of the laboratory, the growth of the past twelve years, has been designed to fit the plan of instruction, according to which the student in psychology may, the first year, attend a course of lectures in which a rich collection of illustrative material is used and experiments are performed before the class by the instructor. The second year, the student may himself perform a series of model experiments, and for the third and following years he may engage in the investigation of original problems. Accordingly the equipment falls into three classes: (1) apparatus, charts and other material for use in class demonstration for the first year in general psychology; (2) a complete set of apparatus for standard exercises which constitute the laboratory course for the second year; and (3) apparatus and the varied means employed in the research work, for special tests, and for advanced demonstration experiments.







